

## Fed Funds Futures Probability Tree Calculator

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CME Group FedWatch uses a combination of single and joint analysis of Federal Open Market Committee (FOMC) meetings, and a probability of a rate change specific to each meeting to generate the binary probability tree. CBOT 30-Day Federal Funds (FF) futures contract prices, which reference the daily federal funds effective rate (FFER) published by the Federal Reserve Bank of New York, are the inputs. This implementation assumes that the size of a rate change is always 25 basis points, and that successive (FF) futures contract prices will be used.

### Next FOMC Meeting

Two probabilities are calculated using the FF contract price of the calendar month following the calendar month of the next FOMC meeting. The calculation assumes that the FOMC's new federal funds target rate (FFTR), will be either a 25-bps increase or a 25-bps decrease from the current FFTR.

The probability of FFTR being set to the lower end of this postulated range of outcomes is the difference of FFTR greater than FFER and the rate given by the FF contract price, divided by the 25-bps possible change. The probability of FFTR being decreased is:

$$\text{Probability}(FFTR\ decr_{t1}) = \frac{((100 - p_d) - (100 - p_{t1+1}))}{0.25}$$

$p_{t1+1}$  = next month's FF contract price

$p_d$  =  $p_{t1+1}$  rounded down to nearest 0.25 increment

e.g., if

$$p_{t1+1} = 98.805$$

$$p_d = 98.75$$

$$\begin{aligned} \text{Probability}(FFTR\ decr_{t1}) &= \frac{((100 - p_d) - (100 - p_{t1+1}))}{0.25} \\ &= \frac{(1.25 - 1.195)}{0.25} \\ &= 22\% \end{aligned}$$

Similarly, the probability of FFTR being set to the higher end of the postulated range of outcomes is the difference of FFTR less than FFER and the rate given by the FF contract price, divided by the 25-bps possible change. The probability of FFTR being increased is:

$$\begin{aligned} \text{Probability}(FFTR\ incr_{t1}) &= \frac{((100 - p_{t1+1}) - (100 - p_u))}{0.25} \\ p_u &= p_{t1+1} \text{ rounded up to nearest 0.25 increment} \end{aligned}$$

e.g., if

$$p_{t1+1} = 98.805$$

$$p_u = 99.00$$

$$\begin{aligned} \text{Probability}(FFTR\ incr_{t1}) &= \frac{((100 - p_{t1+1}) - (100 - p_u))}{0.25} \\ &= \frac{(1.195 - 1)}{0.25} \\ &= 78\% \end{aligned}$$

These two probabilities and possible FFTRs are the start of the binary tree, which determines the range of possible rates and probabilities, at subsequent FOMC meetings.

From the FF prices used in the example, FFER is in between 1% and 1.25%. The probability that the FOMC will lower FFTR to 1.00% is 22%. Alternatively, the probability of the FOMC raising FFTR to 1.25% is 78%.

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### Subsequent FOMC Meetings

Finding the post-meeting implied daily FFER is the first step in calculating the implied probability for various FFTR outcomes at subsequent FOMC meetings. The calculation uses either a joint meeting or single meeting process. In the case of a single meeting, or when the next following month does not contain an FOMC meeting, the FF contract price of the next following month is used. In the case of a joint meeting, or when the next calendar month of the FOMC meeting also contains an FOMC meeting, the current month's FF contract price is used. When a joint meeting occurs, the following month's FF contract price contains market expectations for two FOMC decisions, and so cannot be used for the first meeting.

For a single meeting, the implied rate is:

$$r_{implied} = \frac{(100 - p_{t2+1})N - (100 - p_{t2})(N - M)}{M}$$

$p_{t2+1}$  = next month's FF contract price

$p_{t2}$  = current month's FF contract price

$N$  = number of days in the month

$M$  = day of the month on which FOMC meets

Consider for example an FOMC meeting that occurs on the 16<sup>th</sup> of some 30-day month:  $N = 30$  and  $M = 16$ . Assume that  $p_{t2} = 98.805$  and  $p_{t2+1} = 98.795$  (also note that in this example,  $p_{t2} = p_{t1+1}$  but this will only occur if the prior meeting can be defined as a joint meeting).

$$\begin{aligned} r_{implied} &= \frac{(100 - 98.795)30 - (100 - 98.805)(30 - 16)}{16} \\ &= 1.21375 \end{aligned}$$

For a joint meeting, the implied rate is:

$$r_{implied} = \frac{(100 - p_{t2})N - (100 - p_{t2-1})(M - 1)}{(N - M + 1)}$$

$p_{t2-1}$  = prior month's FF contract price

From the implied post-meeting rate, the probability of a change in FFTR is calculated. The probability is assumed to be independent of the level of the possible effective rate at that time.

The probability of a rate increase is:

$$Probability(rate\ change\ +) = \frac{(r_{implied} - (100 - p_{t2-1}))}{0.25}$$

e.g., if  $r_{implied} = 1.21375$ ,  $p_{t2-1} = 98.82$

$$\begin{aligned} Probability(rate\ change\ +) &= \frac{(1.21375 - (100 - 98.82))}{0.25} \\ &= 13.5\% \end{aligned}$$

The given order of  $r_{implied}$  and  $(100 - p_{t2-1})$  applies when the expectation is that FFTR will be raised, hence the use of *Probability(rate change +)*. If the expectation is for the rate to be cut, the order should be reversed and would be:

$$\mathbf{Probability(rate\ change\ -) = \frac{((100 - p_{t2-1}) - r_{implied})}{0.25}}$$

Returning to the two initial meeting probabilities, and assuming that the expectation is for FFTR to be raised, then at the second meeting we have the three following probabilities:

The probability of a decreased FFTR at the second meeting is:

$$\mathbf{Probability(FFTR\ decr_{t2}) = Probability(FFTR\ decr_{t1})(1 - Probability(rate\ change\ +))}$$

The probability of an unchanged FFTR at the second meeting is:

$$\mathbf{Probability(FFTR_{t2}) = Probability(FFTR\ incr_{t1})(1 - Probability(rate\ change\ +)) + Probability(FFTR\ decr_{t1})(Probability(rate\ change\ +))}$$

The probability of an increased FFTR at the second meeting is:

$$\mathbf{Probability(FFTR\ incr_{t2}) = Probability(FFTR\ incr_{t1})(Probability(rate\ change\ +))}$$

e.g.,

$$\mathbf{Probability(FFTR\ decr_{t2}) = 22\%(1 - 13.5\%) = 19.03\%}$$

$$\mathbf{Probability(FFTR_{t2}) = 78\%(1 - 13.5\%) + 22\%(13.5\%) = 70.44\%}$$

$$\mathbf{Probability(FFTR\ incr_{t2}) = 78\%(13.5\%) = 10.53\%}$$

Using the prior example prices and rates, the probability of FFTR being decreased to 1% is 19.03%. The probability of it being kept at 1.25% is 70.44%, and the probability of it being increased to 1.50% is 10.53%.

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For the third FOMC meeting from now, these three probabilities, a new implied post-meeting rate, and the corresponding probability of a rate change are combined to generate the probabilities of the four possible FFTRs. The process is repeated for each meeting up through the eighth meeting.

**If you have questions, please contact John Wiley at 312-648-3768**

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