An Approach to Compare Exchange-Traded and OTC Option Valuations

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Abstract While OTC and listed option valuations are fundamentally the same, they have four key differences that make it hard to compare the two markets: quoting currency pair convention, value date, underlying instrument, and quotation term. This paper examines a new approach for converting exchange traded vanilla currency options premiums to OTC-equivalent implied volatilities (IV). Moreover, we compare CME OTC-equivalent IV to OTC currency option IV at the standard 4 p.m. London cut. An empirical study shows that the two markets are aligned about two-thirds of the time, confirming their strong relationship. Additionally, we demonstrate that this new conversion assumption holds by verifying that markets' alignment doesn't depend on the distance from option expiry to the underlying futures maturity. Finally, empirical results give us insights into the two markets' relative behaviors: (i) CME volatility wings show excess volatility premium; (ii) CME and OTC IV levels are comparable with the largest differences at short expiries and on extreme deltas; and (iii) cointegration test of standard option strategies shows small p-values, rejecting the hypothesis that there is no cointegrating relationship. It proves that the two markets are reverting to each other and it may be possible to build simple no-arbitrage trading strategies.

Keywords OTC FX option · CME FX option · Futurization · Cointegration · Markets' alignment

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1 Introduction

Vanilla Foreign Exchange Options (FXO) are traded in the over-the-counter (OTC) market and on listed derivatives exchanges. OTC markets are bespoke markets where trading occurs mostly by voice, with $297 billion of daily average turn-over in April 2019, including $156 billion for vanilla G6 FXO[1]. For the main G6 FX currency pairs: EURUSD, JPYUSD, GBPUSD, CADUSD, CHFUSD and AUDUSD, the CME Group's Globex electronic trading system offers a liquid and transparent FXO central limit order book (CLOB) pricing with an average daily volume of $6.3 billion reported in April 2019[2]. While only 4% of the total FX option vanilla market is traded on CME, it is the most traded and liquid CLOB venue, with price discovery available 23 hours a day, 5 days a week. From a trading point of view, both CME and the OTC option markets allow access to similar FX option exposures. However, the two markets have large structural differences. For example, they differ in the way that trades are quoted and executed, in the type of market participants, and in the size of standard trades. Comparing the two markets' option valuations across time, moneyness and expiry would be beneficial to understand if the large structural differences have an impact on the FX option valuation. Unfortunately, there is not enough literature on the study of OTC and listed FXO markets, which compares the two. We study the difference and the alignment of the two markets in this paper. In order to compare the two markets valuations, we first look at how we can make them equivalent. OTC and listed option contracts have four key differences, making it hard to compare the two markets visually without making adjustments. We call the process of removing the four differences the *conversion* process, which transforms CME option premium to OTC-equivalent premium. Without proper conversion the two markets would not be comparable. Finally, we compare the CME converted and OTC IVs to answer three questions:

- What evidence can we find on valuation alignment of the two markets? How often does CME mid-volatility fall inside the OTC spread range, and OTC mid-volatility fall inside the CME spread range?
- Is there a valuation bias (i.e. CME being at a higher option valuation) ?
- Statistically, do we see significant co-integration in the time series of OTC and CME FXO premiums, demonstrating that simple pairs trading strategies can be executed between the markets?

Although many recent books and literature have covered the study of option valuation ([4], [9]) and its applications to the OTC FXO market ([6], [8], [11],[14], [22], [23], [25]), those references have not discussed the differences or similarities between the OTC and listed FXO markets. Literature on the valuation of listed FX option markets are similar ([1], [2], [3], [5], [10], [16], [17], [18], [19], [20], [21], [24]). While providing fundamentals for the pricing of listed options, they do not compare the two markets. In fact, very few papers

[1] Source: BIS Triennial Survey, www.bis.org/statistics/rpfx19.htm

[2] Source: CME Group

have compared the two markets. Cincibuch [7] uses the CME JPYUSD option data to infer the risk neutral density of the OTC FXO market for the purpose of monitoring foreign exchange market and enhancing value-at-risk modeling used by central banks. Cincibuch defines three conversions needed to go from CME option premiums to OTC-equivalent risk neutral distributions: American to European option style[3], underlying futures to spot adjustment, and JPY underlying currency inversion.

Both CME and OTC FXO markets provide exposure to the similar volatility of the underlying FX market. In fact, strong no-arbitrage relationships exist between the two markets for both their underlying instruments and the way the option contracts are defined. First, CME and OTC underlyings, FX futures contracts and FX spot respectively, have a no-arbitrage relationship. The relationship is called *arbitrage-free futures prices.* In practice, the relationship is expected to hold because it is observable, and can be executed directly on the CME CLOB using FX Link which is the listed contract equivalent of an OTC FX swap. FX Link provides a central limit order book on the CME Globex platform for trading spreads between OTC FX and CME FX futures, connecting the two markets to remove any potential arbitrage between spot and futures. Moreover, on its last trading day, FX futures are equivalent and fungible with OTC spot, as they follow the settlement timing and the primary physical delivery mechanism used by the OTC market. Second, CME and OTC option contracts are both European-style[4] option instruments. Additionally, options on the same expiry date share similar value dates[5]. While futures are marked-to-market daily, and profit and loss for CME options are realized T+1, some OTC options have a value date at T+1[6] and others at T+2[7]. For those at T+2, we can make a small time value adjustment, however in very low interest rate environment, this adjustment can be assumed away with minimal impact. Finally, CME cutoff time[8] at expirations is aligned with OTC's 10 am New York convention[9]; consequently, time-to-expiry is identical for options of the same expiry. It is important to note that CME only recently changed the cutoff time from 3 pm to 10 am NY time. CME started the change with long-dated options in June 2018 and progressively modified all option contract listings up to June 2019. In fact, in Section 3.1, we show empirical evidence that the previous 3 pm New York time listing was creating an implied volatility basis

[3] CME changed the option to European-style in 2016.

[4] A European-style option may be exercised only at the expiration date of the option.

[5] Value date refers to the date agreed between the two parties for the mutual delivery of the funds. It is usually one or two business days after the expiry date of the option contract.

[6] For G6, only USDCAD.

[7] All other G6 currency pairs.

[8] CME calculates a volume-weighted average price (VWAP) fixing value on the underlying futures between 9:59 and 10 am NY and it gets published nearly instantly at 10 am. For calls, anything at-the-money or in-the-money against this fixing is exercised, for puts only in-the-money is exercised.

[9] The OTC process requires the option buyer to call its counterpart if he wants to exercise the option.

premium, with the 1-day CME option being always[10] more expensive than the OTC's IV because CME options had 5 more hours of time value.

In summary, OTC and listed FX options are fundamentally aligned and given simple *conversions*, as defined in Section 2, an option on a futures contract is essentially equivalent to an OTC option. Moreover, using the CME OTC-equivalent premium, we compare the two markets in Section 3.

In this paper, we contribute to the literature in several aspects. First, by discussing the different approaches for converting options on futures to its OTC-equivalent and their underlying assumptions by extending the work of Cincibuch [7]. Second, by providing a clear comparison of CME and OTC FXO markets during the period from June 17, 2019 though February 28, 2020 for the most liquid pairs, EURUSD and USDJPY.

2 Conversions

CME FX options have specific discrete strikes and expiry dates in order to concentrate liquidity and fungibility. CME option premiums are traded on the Globex central limit order book which provides full price transparency through the day. All CME FX option contracts expire into the closest quarterly futures contract. Hence, the time-to-maturity of the underlying futures can vary from a few days to almost 3 months. CME FX options used to be American-style when introduced in the 1990's. Their listing was changed in 2016 to European-style to align with the OTC FXO instruments. Because they are listed on an exchange, CME options have specific product codes, expiry period codes, and standardized strike price listings. For example, 'JPUN0 C0935' is the JPYUSD European July 2020 call option expiring in July 2020 with a strike price of 0.00935. The option price is expressed in US dollars per unit of Japanese Yen, for example $0.000093 per 1 Yen, for the contract above.

OTC FX option are typically European-style vanilla options. Their underlying is the spot exchange rate and they are traded in implied volatility terms at pre-defined deltas and fixed tenors, for example, 1-day, 1-week or 1-month. The OTC market is less transparent as trades are negotiated on a bilateral basis by voice, and on many different venues. The difference in terminology also makes it more difficult to try comparing listed versus OTC options. For instance, a listed 'JPUN0 C0935' contract may be the equivalent of what is being referred to as the '1-month USDJPY 25 Delta Put' on the OTC market and the option valuation expressed in implied volatility terms, for instance, 7.15%. Both option contracts above are comparable from a time-to-expiry and moneyness point of view. But which one is better to buy or to sell? How can we compare the CME and OTC FXO market valuations given their differences?

In the OTC market, the convention is to represent tradable volatilities in a grid, as presented in Figure 1. The grid has standard time-to-expiry on the y-axis and delta on the x-axis, and simplifies the way one views FX volatility

[10] 100% of the time.

surface potential opportunities. Usually, the trader can change the grid directly in the trading system interface depending on what he wants to see. For example, the trader can go from a view presenting risk reversals and butterfly volatilities to one showing call/put breakdown by delta, or he can look at two different pairs of attributes: either the mid and the bid-ask spread or the bid and ask volatilities. Figure 1 shows the OTC standard format with call/put view with mid and spread attributes.

Exp\Delta	ATM		35D call EUR		35D put EUR		25D call EUR		25D put EUR		15D call EUR		15D put EUR		10D call EUR		10D put EUR	
	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S
1D																				
1W																				
2W																				
1M																				
2M																				
3M																				
4M																				
6M																				
9M																				
1Y																				

Fig. 1: OTC standard grid format with call/put view and mid (M) and spread (S) attributes.

In order to simplify the comparison between OTC and CME volatilities, we propose to create the same standard grid format for CME OTC-equivalent volatilities. Once the two markets are on the same standard grid, it is straightforward to compare the respective valuations.

Now that we have set up a proper comparison goal, we investigate in detail what needs to be converted. The two markets (CME and OTC) have four key structural differences in product attributes:

1. Quoting currency pair convention
2. Value date
3. Underlying instrument
4. Quotation term

The key differences and the conversions needed are presented in Figure 2; Sections 2.1, 2.2, and 2.3 cover the conversion details for each of the four attributes. It is important to notice that three out of the four conversions are *engineered* conversions, meaning that the assumptions behind the methodology are justified by either simple mathematical pricing equivalence, for the FX underlying inversion, or by industry convention, for the FX pricing model and its adjustment for value date. Only the third conversion requires making a *modeling* assumption. In this particular case, we need to make an assumption on the interest rate dynamics. We derive multiple ways this conversion can be formulated in Section 2.3. The assumption will go over extensive testing in the empirical analysis in Section 3.3.

Conversion #	Product Attribute	CME Listed	OTC Equivalent	Conversion
1	Quoting Currency Pair convention	Quoting currency is USD (JPYUSD)	Quoting currency is SPOT convention (USDJPY)	FX underlying inversion
2	Value date	T+1; CME calendar	T+1 or T+2; Spot Rule	Pricer adjustment for value date
3	Underlying	Future	Spot	Strike adjustment given future to spot basis
4	Quotation term	Price	Volatility	Garman–Kohlhagen FX pricing model

Fig. 2: Summary of key differences between CME and OTC markets product attributes and required conversion.

2.1 Quoting Currency Pair Conversion

The first difference between CME and OTC markets is the quoting convention. CME FX futures contracts are listed in American terms with US dollars as the quoting currency[11]. While the OTC market uses the underlying spot convention which can be either quoted in American terms or quoted in non-American terms depending on the spot market convention[12] as defined by Clark [8]. When the OTC quoting convention is not using American terms, it is needed to convert the observed CME option premium V_{XXXUSD} into an OTC equivalent option premium V'_{USDXXX}. Let's call this process the *FX underlying currency pair inversion*. One can replicate the payoff of buying XXXYYY call (put) option with strike price K and maturity T, by buying K units of YYYXXX put (call) option with strike price 1/K and maturity T. Figure 3 presents two option positions with identical payoff.

	VALUATION	OPTION TYPE	POSITION	NOTIONAL	CURRENCY PAIR	STRIKE	TIME TO MATURITY	PAYOFF GIVE	PAYOFF RECEIVE
CME	V	Call	Long	100	JPYUSD	1/100	1M	1 USD	100 JPY
OTC	V′	Put	Long	1	USDJPY	100	1M	1 USD	100 JPY

Fig. 3: FX underlying currency pair inversion on JPY for CME and OTC.

We define mathematically V' in Equation 2.1.

$$V'_{\mathrm{YYYXXX}}(\Omega' = 1 - \Omega, F' = \frac{1}{F}, K' = \frac{1}{K}) = \frac{V_{\mathrm{XXXYYY}}(\Omega, F, K)}{K} S_{\mathrm{YYYXXX}} \quad (2.1)$$

Where $\Omega = 0$ for the call option, $\Omega = 1$ for the put option and S is FX spot; both options have the same expiry date and and underlying delivery date (value date). Cincibuch uses the same conversion in his paper [7]. Note that this process changes the sign of the risk reversal which should be addressed.

[11] General notation, XXXUSD. For examples, JPYUSD or EURUSD.

[12] For example, USDJPY is in Japanese term and USDCAD is in Canadian terms.

2.2 Price Adjustment for Value Date

The second conversion is the adjustment of the value date from CME to OTC in the pricer. An excercised option is settled on option delivery date, also called *value date*. The OTC option value date is by convention 1 or 2 days after option expiry date. The OTC convention uses the *Spot Rule* (see Clark [8]) to count days between the expiry date (T_{exp} or T) and the value date. In FXO jargon, these days are referenced as T+1 and T+2. The OTC option's exercise involves the exchange of a specified, fixed notional amount on the value date. The CME option's exercise is slightly different due to payment timelines. Because the CME option expires into a futures contract, option holders receive the underlying futures position and the option profit and loss at T + 1. CME uses a CME calendar to count 1 day from expiry to value date. Figure 4 summaries the T+1 or T+2 rule between the CME and OTC option value date calculations for five currency pairs.

Currency pair (OTC convention)	*OTC*	*CME*
EURUSD	T+2	T+1
USDJPY	T+2	T+1
AUDUSD	T+2	T+1
GBPUSD	T+2	T+1
USDCAD	T+1	T+1

Fig. 4: T+1 or T+2 rule between the CME and OTC option value date calculations.

For USDCAD, if the CME calendar and the spot rule match, we have no difference. For the other pairs, the adjustment from T+1 to T+2 has a very small impact on the Garman–Kohlhagen FXO pricer ([11]) because we are only looking at a 1-day discount factor mismatch. Figure 5 shows the assumption made in the adjustment. Because the discount factor price adjustment from the CME payment date to the OTC delivery dates is small for G10 currencies, we chose to ignore it in the below formula.

	PAYMENT DATE	PAYMENT DATE – REMOVE 1-DAY DISCOUNTING	PAYMENT
CME	Expiry Date +1	Expiry Date +1 or +2	P&L + ATM Future
OTC	Expiry Date +1 or +2	Expiry Date +1 or +2	Notional Exchange

Fig. 5: : Payment comparison between CME and OTC.

If we do not want to ignore the small difference in the discount factor, it is possible to apply the conversion by using the OTC Garman–Kohlhagen FXO pricer on CME data when converting from price to volatility.

2.3 Option Underlying Instrument Conversion

The third conversion is the adjustment in the underlying instrument from futures to spot. As defined in Figure 5, there is a difference in the payoff between CME and OTC options. The former delivers profit and loss and a futures, and the latter delivers a payment of notional on the value date. We define V_{OTC}

as the premium of a CME option delivering on the OTC value date (T_{del}) and V_{fut} the premium of a CME option with a futures underlying (expiring on T_2). Both options share the same expiry date T_1 . The two option premiums are a function of each other if we make a small adjustment to the strike price K_{OTC}. There are many ways to write the function and the adjustment will depend on the assumptions we are willing to make and the data input available for the conversion. We start by defining the option on futures call premium with Equation 2.2.

$$V_{\text{fut}}(t, K_{\text{fut}}, T_1, T_2, T_{del}) = E_t[e^{-\int_t^{T_{del}} r(s)ds}(F_{T_1,T_2} - K_{\text{fut}})^+] \tag{2.2}$$

where:

- V_{fut} is the option on futures call premium at time t, strike price K_{fut}, expiry date T_1, and futures maturity date T_2;
- $E_t[.]$ is the risk neutral expectation conditional on the information available at an earlier time t under the risk neutral measure associated with riskless investment in a money market account we denote by $M(t, t, T_{del}) = e^{-\int_t^{T_{del}} r(s)ds}$. $r(s)$ is a short rate process associated with this riskless investment.
- F_{T_1,T_2} is the price of the option underlying futures at time T_1 and maturity T_2.

We further define S_{T_1} to be the FX spot rate at time T_1 and V_{OTC} to be the OTC-equivalent call option premium with strike price K_{OTC}.

$$V_{\text{OTC}}(t, K_{\text{OTC}}, T_1, T_{del}) = E_t[e^{-\int_t^{T_{del}} r(s)ds}(S_{T_1} - K_{\text{OTC}})^+] \tag{2.3}$$

We evaluate four possible methods for addressing the option underlying conversion issue. All four formulations require us to make assumptions on the interest rate dynamics. Practically, the impact of the choice of interest rate dynamics should be limited because the maximum distance between option expiry and futures maturity on CME contracts is 3 months, and the maximum CME option contract time-to-expiry is about 1-year. A *standard*[13] assumption in valuing a short-expiry vanilla FX options is to set the interest rates as non-stochastic. For example, Clark [8] uses the *standard* assumption to derive the valuation of options on forwards[14]. We use the non-stochastic interest rates assumption when deriving Formulations 1 (Section 2.3.1), 2 (Section 2.3.2) and 3 (Section 2.3.3). Formulation 4 (Section 2.3.4) is used by Cincibuch [7] and assumes constant interest rate. From a modeling standpoint, Formulation 4 can be seen as inferior to the first three. Formulation 2 uses the same set of assumptions as Formulations 1 and 3, while making an additional assumption.

[13] And probably a benchmark.
[14] Chapter: delayed delivery adjustment for option on forward.

By letting the interest rates be non-stochastic, the discount factor comes out from the expectation, and we can rewrite Equation 2.4 as:

$$V_{fut}(t, K_{fut}, T_1, T_2, T_{del}) = D^d(t, t, T_{del}) E_t[(F_{T_1,T_2} - K_{\text{fut}})^+] \tag{2.4}$$

where $D^d(t, t, T_{del}) = e^{-\int_t^{T_{del}} r(s)ds}$. We also assume *arbitrage-free futures prices*. That there is an absence of arbitrage between the futures price F_{t,T_1} at the current time t to the expected spot price S_{T_1} at the maturity date T, of the relevant futures contract, conditioned on the information at time t as defined in Equation 2.5 (see Gunther, et al [12]):

$$F_{t,T_1} = E_t[S_{T_1}] \tag{2.5}$$

It is worth mentioning that under the four formulations below the futures price and the forward price will be equivalent. Non-stochastic or constant interest rates would fix interest rate volatility to zero, and zero volatility makes the covariance between FX spot and interest rate null as well, which, in consequence, provides equivalence between futures and forward prices as detailed in Appendix A: $\text{Fwd}_{t,T_1} = F_{t,T_1} = E_t[S_{T_1}]$.

2.3.1 Formulation 1:

$$\begin{aligned} V_{fut} &= D^d(t, t, T_{del}) E_t \left[\left(S_{T_1} \frac{F_{T_1,T_2}}{S_{T_1}} - K_{\text{fut}} \right)^+ \right] \\ &= D^d(t, t, T_{del}) E_t \left[\left(S_{T_1} \frac{F_{t,T_2}}{F_{t,T_1}} - K_{\text{fut}} \right)^+ \right] \\ &= D^d(t, t, T_{del}) \frac{F_{t,T_2}}{F_{t,T_1}} E_t \left[\left(S_{T_1} - K_{fut} \frac{F_{t,T_1}}{F_{t,T_2}} \right)^+ \right] \\ &= \frac{F_{t,T_2}}{F_{t,T_1}} V_{\text{OTC}}(t, K_{\text{OTC}}, T_1, T_{del}) \end{aligned} \tag{2.6}$$

Solving for V_{OTC}, we have:

$$V_{\text{OTC}}(t, K_{\text{OTC}}, T_1, T_{del}) = \frac{F_{t,T_1}}{F_{t,T_2}} V_{\text{fut}}(t, K_{\text{fut}}, T_1, T_2, T_{del}) \tag{2.7}$$

with:

$$K_{\text{OTC}} = K_{\text{fut}} \frac{F_{t,T_1}}{F_{t,T_2}} \tag{2.8}$$

2.3.2 Formulation 2:

$$\begin{aligned} V_{\text{fut}} &= D^d(t,t,T_{del})E_t\left[(F_{T_1,T_2} - S_{T_1} + S_{T_1} - K_{\text{fut}})^+\right] \\ &= D^d(t,t,T_{del})E_t\left[(S_{T_1} - (K_{\text{fut}} - \phi))^+\right] \\ &= V_{\text{OTC}}(t, K_{\text{OTC}}, T_1, T_{del}) \end{aligned} \tag{2.9}$$

with:

$$K_{\text{OTC}} = K_{\text{fut}} - \phi \tag{2.10}$$

and:

$$\phi = F_{T_1,T_2} - S_{T_1} \tag{2.11}$$

However, because F_{T_1,T_2} and S_{T_1} are unknown at time t, we need to find an approximation for the term. Instead of evaluating ϕ, we decide to look at $E_t[\phi]$ and define as ϕ'.

$$\begin{aligned} E_t[\phi] &= E_t[F_{T_1,T_2} - S_{T_1}] \\ &= E_t[F_{T_1,T_2}] - E_t[S_{T_1}] \\ &= E_t[E_{T_1}[S_{T_2}]] - \text{Fwd}_{t,T_1} \\ &= E_t[S_{T_2}] - \text{Fwd}_{t,T_1} \\ &= F_{t,T_2} - \text{Fwd}_{t,T_1} \\ &= \phi' \end{aligned} \tag{2.12}$$

Because we assume arbitrage-free futures prices, we can write:

$$\begin{aligned} E_t[F_{T_1,T_2}] &= E_t[E_{T_1}[S_{T_2}]] \\ E_t[S_{T_2}] &= F_{t,T_2} \end{aligned} \tag{2.13}$$

Although this is not true in the real world that $\phi = \phi'$, we expect the error to be small for an option with time-to-maturity less than one year. Thus, in practice we will use Equation 2.14 instead of Equation 2.10 to calculate K_{OTC}.

$$K_{\text{OTC}} = K_{\text{fut}} - \phi' \tag{2.14}$$

2.3.3 Formulation 3:

$$\begin{aligned} V_{\text{fut}} &= D^d(t,t,T_{del})E_t\left[\left(S_{T_1}\frac{F_{T_1,T_2}}{S_{T_1}} - K_{\text{fut}}\right)^+\right] \\ &= D^d(t,t,T_{del})E_t\left[\left(S_{T_1}\frac{D^f(t,T_1,T_2)}{D^d(t,T_1,T_2)} - K_{\text{fut}}\right)^+\right] \\ &= D^d(t,t,T_{del})\frac{D^f(t,T_1,T_2)}{D^d(t,T_1,T_2)}E_t\left[\left(S_{T_1} - K_{\text{fut}}\frac{D^d(t,T_1,T_2)}{D^f(t,T_1,T_2)}\right)^+\right] \\ &= \frac{D^f(t,T_1,T_2)}{D^d(t,T_1,T_2)}V_{\text{OTC}}(t, K_{\text{OTC}}, T_1, T_{del}) \end{aligned} \tag{2.15}$$

Solving for V_{OTC}, we have:

$$V_{\text{OTC}}(t, K_{\text{OTC}}, T_1, T_{del}) = \frac{D^d(t, T_1, T_2)}{D^f(t, T_1, T_2)} V_{\text{fut}}(t, K_{\text{fut}}, T_1, T_2, T_{del}) \tag{2.16}$$

with:

$$K_{\text{OTC}} = K_{\text{fut}} \frac{D^d(t, T_1, T_2)}{D^f(t, T_1, T_2)} \tag{2.17}$$

2.3.4 Formulation 4:

Equations 2.19 and 2.20 are, respectively, equivalent to Equations (3) and (4) in Cincibuch [7].

$$\begin{aligned} V_{\text{fut}} &= D^d(t, t, T_{del}) E_t \left[\left(S_{T_1} \frac{F_{T_1,T_2}}{S_{T_1}} - K_{\text{fut}} \right)^+ \right] \\ &= D^d(t, t, T_{del}) E_t \left[\left(S_{T_1} e^{(r_d - r_f)(T_2 - T_1)} - K_{\text{fut}} \right)^+ \right] \\ &= D^d(t, t, T_{del}) e^{(r_d - r_f)(T_2 - T_1)} E_t \left[\left(S_{T_1} - e^{-(r_d - r_f)(T_2 - T_1)} K_{\text{fut}} \right)^+ \right] \\ &= e^{(r_d - r_f)(T_2 - T_1)} V_{\text{OTC}}(t, K_{\text{OTC}}, T_1, T_{del}) \end{aligned} \tag{2.18}$$

Solving for V_{OTC}, we have:

$$V_{\text{OTC}}(t, K_{\text{OTC}}, T_1, T_{del}) = e^{-(r_d - r_f)(T_2 - T_1)} V_{\text{fut}}(t, K_{\text{fut}}, T_1, T_2, T_{del}) \tag{2.19}$$

with:

$$K_{\text{OTC}} = e^{-(r_d - r_f)(T_2 - T_1)} K_{\text{fut}} \tag{2.20}$$

In summary, Formulation 4 makes the strongest assumptions, i.e. constant interest rate, and should preferably not be used for conversion. Formulation 2 approximates the adjustment amount, resulting in a small error of converted option and strike prices. Finally, the first three formulations should have an acceptable assumption as they follow the industry *standard* for short dated FX options.

How do Formulations 1, 2, and 3 compare from an implementation standpoint? Formulation 1 and 2 require the modeling of the futures (or forward) curve, while Formulation 3 requires the modeling of foreign and domestic interest rate curves. Formulations 1 and 2 need bootstrapping of the futures curve at expiry dates T_1. To avoid any extrapolation on the short-dated maturity, we should bootstrap the futures curve by using FX spot as first anchor point. To our knowledge, bootstrapping the futures curve is not a standard practice. However, it is possible to change the future curve by the forward curve for formulation 1 and 2. It is easier to implement as it requires bootstrapping of the forward curve, a standard practice in OTC FX. We implemented both

Formulations 1 and 2 using futures and forward curves and they produce similar empirical results. Thus, we decide to use Formulation 2 in the rest of the document and in the empirical analysis Section 3.

With V_{OTC} defined, we can solve for the fourth structural difference and use the standard Garman–Kohlhagen pricing model to back out volatility $\sigma(K_{\text{OTC}}, T_1)$ from V_{fut} as defined in Clark [8].

3 Empirical Analysis

In this section, we perform an empirical analysis using the converter on EURUSD and JPYUSD currency pairs. In Section 3.1, we demonstrate that CME data prior to June 17, 2019 cannot be used for the comparison. It contains a premium bias due to the CME option having a cutoff 5 hours later than the OTC market. Then, in Section 3.2, we define a measure of *alignment of the markets*. In Section 3.3, we test empirically the converter non-stochastic interest rate assumption of Formulation 2 (2.3.2) using this measure. Finally, in Section 3.4, we verify the performance of the conversion and compare the OTC and CME FX option markets.

For the analysis, we compare raw data from CME Globex, previously converted to OTC-equivalent volatility, as detailed in Section 2, to OTC volatilities from June 17, 2019 through February 28, 2020. The OTC data used as a benchmark is sourced from high quality credible sources, and is an aggregate of several important OTC FX option market participants at the 11 am NY time snap[15]. The data is of superior quality given the market presence of the sources and especially because the vendor verifies daily the output by assessing quantitative and qualitative measures (i.e. outlier detection, rejection of contributor, statistics of change versus historical data, performance of calibration, etc.). The data output includes mid-volatility and one standard-deviation values for each of the 9 moneyness points representing deltas of $\{10, 15, 25, 35, 50, 65, 75, 85, 90\}$ and all expiries: 1-day, weeklies from 1 to 12, monthlies from 4 to 9, and finally semi-annual from 1-year to 2-year.

For comparison purposes, we align the OTC-equivalent CME Globex data to the OTC data. We use the same snap time, same moneyness, and same expiries as the OTC data. The snap time is the 11 am NY time and can be easily gathered from the CME Globex CLOB data, the moneyness is matched by fitting SABR[16] to CME OTC-equivalent option time values, and the expiries are made identical by removing any CME option that does not match OTC expiries. As such, time-interpolation of volatility is required for the comparison. Finally, the expiry time cutoffs for CME and OTC options match because we use data after June 2019.

In order to validate the conversion process and compare the CME and OTC markets, we want to use mid-volatilities as well as bid and ask data. CME Globex CLOB aggregates in real time the best bid and best ask of the

[15] Identical to the 4 pm London snap time most of the year.

[16] Stochastic Alpha Rho model, Hagan, et al. [13].

option premiums of limit orders from all market participants. Thus, CME data provides observed best bid and best ask option premiums from which the mid-premium can be implied. As such, the reported mid-volatility is the mid-premium, converted to the OTC-equivalent and which was moneyness-interpolated using SABR as described in Appendix C. Tables 6 and 7, show, respectively, the bid and ask of CME FX option data from CME Globex as of September 30, 2019.

	10C	15C	25C	35C	50C	35P	25P	15P	10P
1D	5.97%	5.84%	5.73%	5.69%	5.72%	5.83%	5.98%	6.25%	6.49%
1W	5.52%	5.36%	5.19%	5.14%	5.18%	5.34%	5.54%	5.89%	6.18%
2W	5.58%	5.39%	5.20%	5.12%	5.13%	5.27%	5.47%	5.81%	6.08%
3W	5.69%	5.52%	5.35%	5.28%	5.29%	5.42%	5.59%	5.90%	6.16%
4W	5.93%	5.79%	5.63%	5.58%	5.59%	5.70%	5.86%	6.13%	6.36%
5W	6.05%	5.90%	5.76%	5.71%	5.73%	5.84%	5.98%	6.25%	6.48%
6W	6.04%	5.88%	5.71%	5.64%	5.65%	5.77%	5.93%	6.22%	6.46%
7W	6.04%	5.87%	5.70%	5.63%	5.64%	5.76%	5.92%	6.21%	6.46%
8W	6.04%	5.87%	5.69%	5.62%	5.63%	5.74%	5.91%	6.20%	6.45%
9W	6.04%	5.87%	5.68%	5.61%	5.62%	5.73%	5.90%	6.19%	6.44%
10W	6.06%	5.88%	5.69%	5.61%	5.62%	5.73%	5.90%	6.20%	6.45%
11W	6.09%	5.91%	5.71%	5.63%	5.64%	5.75%	5.92%	6.23%	6.48%
12W	6.12%	5.93%	5.73%	5.65%	5.65%	5.77%	5.93%	6.24%	6.51%
4M	6.23%	6.03%	5.81%	5.72%	5.71%	5.83%	6.00%	6.33%	6.60%
5M	6.31%	6.09%	5.85%	5.76%	5.75%	5.86%	6.04%	6.37%	6.66%
6M	6.44%	6.21%	5.97%	5.86%	5.85%	5.96%	6.14%	6.48%	6.77%
9M	6.57%	6.27%	5.94%	5.79%	5.76%	5.89%	6.11%	6.53%	6.88%
1Y	6.81%	6.47%	6.07%	5.89%	5.84%	6.00%	6.25%	6.73%	7.13%

Fig. 6: EURUSD CME bid volatility data on September 30, 2019 sourced from CME Globex. Snap is at 11 am NY.

In comparison, OTC participants provide mid volatility surface data daily to the OTC FX option data vendor. However, contributing OTC participants' bid and ask volatilities are unknown. The vendor aggregates the participants' information into mid-volatility and standard deviation of volatilities. Tables 8 and 9 show, respectively, the mid and standard deviations of OTC FX option data as of September 30, 2019. It is a market standard for data aggregation venues to share output mid and standard deviation of the inputs. In a normal distribution, the range $[\mu-2\sigma, \mu+2\sigma]$ represents 95.45% of the data. Assuming that the deviation in submission between OTC FX option market participants is Gaussian, such range would capture most the submissions. Thus, any buy-side client executing on the OTC market can trade with any of the participants within the range. Subsequently, in the following analysis, we use the range as if it was an equivalent representation of an "implied OTC bid-ask spread". It is important to bear in mind that the purpose of this exercise is not to compare the bid-ask spread or liquidity of the two markets. As mentioned above, data on the OTC tradable bid-ask spread is not available to us. Instead, we are

	10C	15C	25C	35C	50C	35P	25P	15P	10P
1D	6.82%	6.71%	6.61%	6.58%	6.60%	6.68%	6.80%	7.02%	7.22%
1W	6.09%	5.91%	5.71%	5.63%	5.64%	5.77%	5.95%	6.29%	6.57%
2W	5.94%	5.75%	5.54%	5.45%	5.46%	5.60%	5.80%	6.15%	6.44%
3W	5.99%	5.80%	5.60%	5.52%	5.52%	5.66%	5.85%	6.18%	6.46%
4W	6.22%	6.04%	5.86%	5.79%	5.79%	5.91%	6.08%	6.38%	6.63%
5W	6.28%	6.12%	5.95%	5.89%	5.90%	6.02%	6.18%	6.47%	6.71%
6W	6.22%	6.04%	5.86%	5.79%	5.80%	5.92%	6.09%	6.39%	6.64%
7W	6.22%	6.03%	5.84%	5.76%	5.77%	5.89%	6.06%	6.38%	6.64%
8W	6.21%	6.02%	5.82%	5.74%	5.74%	5.87%	6.04%	6.36%	6.62%
9W	6.21%	6.02%	5.81%	5.73%	5.73%	5.85%	6.02%	6.34%	6.60%
10W	6.23%	6.03%	5.81%	5.73%	5.72%	5.84%	6.01%	6.34%	6.61%
11W	6.27%	6.06%	5.84%	5.75%	5.74%	5.86%	6.04%	6.37%	6.65%
12W	6.30%	6.09%	5.86%	5.77%	5.76%	5.88%	6.06%	6.39%	6.68%
4M	6.43%	6.20%	5.96%	5.85%	5.84%	5.95%	6.13%	6.47%	6.76%
5M	6.48%	6.24%	5.98%	5.88%	5.86%	5.98%	6.16%	6.51%	6.80%
6M	6.64%	6.38%	6.10%	5.98%	5.96%	6.08%	6.28%	6.65%	6.97%
9M	6.97%	6.71%	6.43%	6.32%	6.30%	6.41%	6.60%	6.97%	7.29%
1Y	7.18%	6.88%	6.54%	6.40%	6.36%	6.50%	6.71%	7.14%	7.50%

Fig. 7: EURUSD CME ask volatility data on September 30, 2019 sourced from CME Globex. Snap is at 11 am NY.

making a reasonable assumption of what an OTC bid-ask spread might be given our data, and with it we are validating the converter performance.

	10C	15C	25C	35C	50C	35P	25P	15P	10P
1D	6.23%	6.09%	5.96%	5.92%	5.95%	6.06%	6.23%	6.54%	6.78%
1W	5.54%	5.46%	5.36%	5.30%	5.31%	5.44%	5.61%	5.89%	6.09%
2W	5.57%	5.44%	5.32%	5.26%	5.27%	5.39%	5.55%	5.83%	6.05%
3W	5.64%	5.51%	5.37%	5.29%	5.29%	5.43%	5.60%	5.88%	6.10%
4W	5.86%	5.75%	5.63%	5.55%	5.55%	5.67%	5.83%	6.09%	6.30%
5W	6.07%	5.96%	5.83%	5.77%	5.76%	5.88%	6.04%	6.30%	6.50%
6W	6.01%	5.89%	5.75%	5.69%	5.68%	5.81%	5.96%	6.22%	6.43%
7W	6.02%	5.88%	5.73%	5.67%	5.66%	5.78%	5.94%	6.20%	6.41%
8W	6.01%	5.87%	5.71%	5.64%	5.64%	5.75%	5.91%	6.17%	6.39%
9W	6.01%	5.86%	5.69%	5.62%	5.62%	5.73%	5.88%	6.15%	6.37%
10W	6.06%	5.90%	5.72%	5.64%	5.64%	5.75%	5.91%	6.19%	6.42%
11W	6.25%	6.08%	5.89%	5.80%	5.79%	5.91%	6.09%	6.38%	6.60%
12W	6.26%	6.08%	5.89%	5.79%	5.78%	5.91%	6.08%	6.36%	6.60%
4M	6.33%	6.12%	5.89%	5.79%	5.76%	5.88%	6.06%	6.37%	6.64%
5M	6.41%	6.19%	5.94%	5.83%	5.79%	5.91%	6.10%	6.43%	6.70%
6M	6.53%	6.30%	6.02%	5.89%	5.85%	5.97%	6.16%	6.51%	6.80%
9M	6.74%	6.47%	6.15%	6.00%	5.95%	6.07%	6.27%	6.66%	6.98%
1Y	6.96%	6.68%	6.33%	6.16%	6.09%	6.22%	6.45%	6.87%	7.20%

Fig. 8: EURUSD OTC volatility mid data on September 30, 2019 sourced from OTC data vendor. Snap is at 11 am NY.

Figure 10 shows an example of the time series of 1-week expiry option comparing OTC-equivalent CME mid/bid/ask volatilities and OTC mid and

	10C	15C	25C	35C	50C	35P	25P	15P	10P
1D	0.0023	0.0026	0.0028	0.0028	0.0028	0.0028	0.0028	0.0029	0.0029
1W	0.0010	0.0009	0.0009	0.0009	0.0008	0.0009	0.0009	0.0010	0.0011
2W	0.0011	0.0009	0.0006	0.0005	0.0003	0.0004	0.0005	0.0005	0.0006
3W	0.0005	0.0003	0.0003	0.0004	0.0005	0.0004	0.0004	0.0005	0.0006
4W	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006
5W	0.0005	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0004	0.0004
6W	0.0006	0.0005	0.0004	0.0003	0.0002	0.0003	0.0004	0.0004	0.0005
7W	0.0005	0.0004	0.0003	0.0002	0.0001	0.0001	0.0002	0.0003	0.0003
8W	0.0007	0.0005	0.0004	0.0002	0.0001	0.0002	0.0003	0.0004	0.0005
9W	0.0007	0.0006	0.0004	0.0003	0.0002	0.0003	0.0004	0.0005	0.0005
10W	0.0006	0.0005	0.0004	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005
11W	0.0004	0.0004	0.0003	0.0003	0.0003	0.0002	0.0003	0.0002	0.0002
12W	0.0004	0.0004	0.0004	0.0003	0.0002	0.0003	0.0003	0.0003	0.0003
4M	0.0004	0.0003	0.0004	0.0003	0.0003	0.0003	0.0004	0.0004	0.0004
5M	0.0003	0.0003	0.0004	0.0004	0.0004	0.0004	0.0005	0.0004	0.0004
6M	0.0005	0.0003	0.0002	0.0002	0.0002	0.0003	0.0004	0.0004	0.0006
9M	0.0006	0.0004	0.0003	0.0003	0.0003	0.0004	0.0005	0.0005	0.0007
1Y	0.0006	0.0004	0.0003	0.0003	0.0003	0.0004	0.0004	0.0005	0.0006

Fig. 9: EURUSD OTC volatility one standard deviation on September 30, 2019 sourced from OTC data vendor. Snap is at 11 am NY.

mid $+/-$ 2 standard deviations. The yellow marker indicates when OTC mid-volatility is outside of the CME bid-ask spread range. Similarly, the black marker specifies when CME mid-volatility is outside of the OTC spread range. The bottom graph shows the difference between CME and OTC mid-volatilities and if the CME converted volatility was calculated by temporal interpolation.

Furthermore, to provide accurate statistics on CME versus OTC FX option markets, we only keep data that is truly comparable. Consequently, no time interpolation modeling is used[17], and only data with good enough SABR fit is kept[18], thus ensuring small differences between true mid-market and SABR interpolated volatilities.

A simple data pre-processing method is used to make the algorithm more robust from the CME live data snap. On CME option data, we remove in-the-money options, options with no bid and/or no ask are filtered out and any option with premium less than two ticks. On CME futures data, there are cases where limit order book information for option contracts is available while information for their underlying futures is not. For such cases, we extract underlying futures price information from options assuming that put-call parity holds. This method is used in the OTC S&P 500 option market to imply the (non-traded) underlying forward price (see Joseph [15]). Appendix B covers the process in detail with a numerical example.

[17] To clarify, in the below analysis, CME and OTC option expiry dates are always equal.

[18] Only a dozen expiries have been removed after setting a maximum implied volatility root-mean-squared-error (RMSE) of 0.02 for both currency pairs. All outliers are due to an arbitrageable smile, demonstrating that our data pre-processing methodology is not yet robust. However, it is not the goal of this paper to investigate this challenge so we choose data removal over fixing.

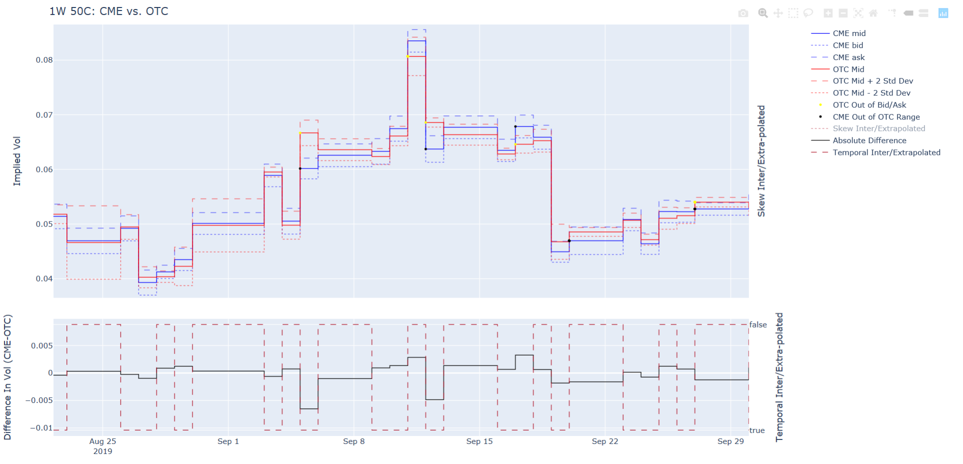

Fig. 10: Time series of EURUSD 1-week expiry option at 50 delta from August 22 to September 30 2019. The top graph shows OTC mid-volatility (red), OTC mid +/- 2 standard deviation (red dash), as well as, OTC-equivalent CME mid-volatility (blue) and CME bid and ask (blue dash). The yellow and black dots, respectively, show when OTC mid-volatility is out of the CME bid/ask spread and when CME mid-volatility is out of the OTC range. The bottom graph presents absolute differences of IVs between CME and OTC markets and the temporal interpolation indicator.

3.1 Data Bias Due to Option Expiry Time Cutoff

In this next section, we investigate data bias in the historical data due to differences of option cutoff times between CME and OTC options.

The most liquid OTC FX option expiry cutoff is 10 am NY and is called the *New York cut*. Prior to July 2018, the CME FX option expiry cutoff was at 3 pm ET, 5 hours later than the OTC standard. CME began to change the listings on the longest-dated options and progressively converted the entire available options, ending with the weekly options in June 2019. Given that CME options had 5 more hours of time value compared to OTC options, we expect to see CME short expiring options at premium[19] to the same-duration OTC options prior and a balanced premium/discount after the option listing change. Furthermore, we expect the impact to be much larger on very short-dated options as 5 hours of extra time is important on a 1-day option but trivial on a 1-year option.

To verify the impact of the cutoff change on the OTC-equivalent CME FX option volatility, we compare it with OTC mid-volatilities prior and after the listing change on 1-day and 1-week expiries. We extract a few months of EURUSD data before and after the change and make the number of data points identical. Figure 11 displays four plots. The dark green color indicates the percentage of time when CME is at premium, with mid-volatility higher

[19] Premium: CME has higher volatility compared to OTC. Discount: CME has lower volatility compared to OTC.

to OTC, and the light green color shows the percentage of time when CME is at discount. The top-right plot indicates that 100% of the 1-day CME option at premium to OTC prior to cutoff at 3 pm ET while the same option has a balanced premium/discount ratio when listed at the same cutoff as OTC on the top left plot. On the bottom left, the 1-week option shows a similar trend, but less pronounced, with around 95% of date being at premium.

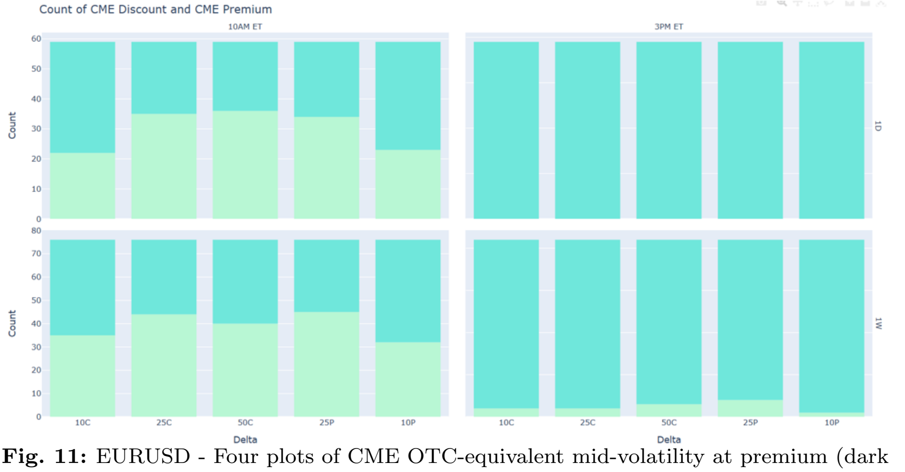

Fig. 11: EURUSD - Four plots of CME OTC-equivalent mid-volatility at premium (dark green) or at discount (light green) to the OTC. The top two graphs show 1-day options, and bottom two, 1-week options. The left two graphs show CME options listed at the 10 am ET cutoff, and the right two, options listed at 3 pm ET.

The results are as expected. CME 1-day options are always at premium compared to their counterparts prior to the listing change. After the option listing change, the premium/discount ratio is balanced. In conclusion, we can see that the change of option listing to the New York cut was an effective modification to ensure a better alignment between CME and OTC FX options.

We have run the converter on CME data from January 2018 up to the end of February 2020. As demonstrated above, short-dated options show abnormal volatility differences between CME and OTC markets (prior to June 2019). To remove any data bias, we decided to only compare the two markets with data from June 2019 to the end of February 2020.

In the next section, we analyze the quality of the FX volatility converter and test the non-stochastic interest rate assumption made in the second Formulation 2.3.2 of the third conversion process (2.3).

3.2 Measure of Alignment of Markets

We define a measure of *alignment of markets* as a way to identify how close both markets are to each other. We compute Boolean indicators that verify:

- if OTC mid is within the CME bid-ask spread,

– if CME mid is within OTC mid +/- 2 standard deviation range.

The *best case* scenario for this measure is when both Booleans are true. In this case, we have high overlap between CME and OTC volatility bid-ask ranges, and we can infer that CME and OTC volatilities are very close to each other. In this case, we can say that both markets are *aligned*. The *worst case* scenario is when both Booleans are false, indicating we have no overlap between the two markets, markets are *not aligned*. The two middle cases, either true/false or false/true, show that CME and OTC volatilities are neither too far nor too close to each other. The measure can be used on all data points to see how markets are aligned to each other. It can also be used on aggregated data by delta or expiration as we will see in Section 3.4 to get an idea on how this alignment change across the volatility surface. Finally, this measure can also be use to test the third conversion's assumption.

3.3 Testing Third Conversion's Assumption

Out of all the conversions needed to adjust CME data to OTC-equivalent volatilities, only the option underlying conversion is an adjustment requiring modeling. As mentioned in 2.3, we decided to use Formulation 2 (2.3.2) which makes the industry standard assumption, for short-dated FX options, of non-stochastic interest rate dynamics.

The more days between option expiry and futures maturity, defined as Γ, the more the interest rate assumption will be put to the test. If the assumption does not hold, we should expect the *best case* to deteriorate and the worst case to improve as Γ increases. Given that all CME FX option contracts expire into the closest quarterly futures contract, the largest option underlying conversion is around 90 days at maximum. In fact, we observe empirically that Γ, is between 10 and 96 days. Consequently, the option underlying conversion (the Γ adjustment) should have a limited impact on the conversion results for the current environment.

From Figure 13, we know that the *best case* averages are 65.0% and 67.4% with the worst case averages 15.5% and 13.8% for EURUSD and USDJPY, respectively. Figure 12 shows the percentage of data points in each of the cases mentioned above as a function of Γ for EURUSD with the best case values ranging between 50% and 83% of the total and the *worst case* ranging between 7% and 30%. The percentage of *best case* does not decrease with Γ with all the long-dated Γ buckets being larger than or at the average. Moreover, the percentage of *worst case* does not decrease with Γ either. Figure 29 shows the same information for USDJPY. On the graph, the best case values range between 40% and 80% of the total, while the *worst case* ranging between 4% and 31%. For USDJPY, the percentage of *best case* seems to increase with Γ with all the long-dated Γ buckets being larger than or at the average and the percentage of *worst case* seems to decrease with Γ.

In conclusion, Figures 12 and 29 show empirically that the alignment between the markets is not decreasing as Γ increases, demonstrating that the

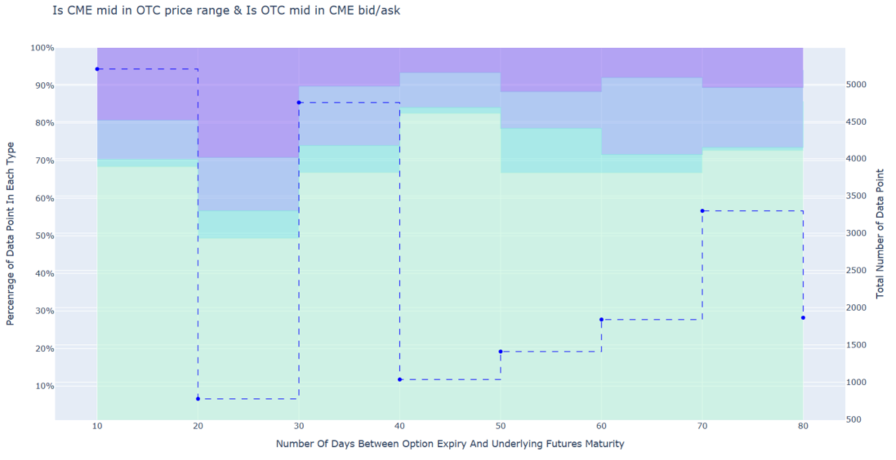

Fig. 12: EURUSD - Percentage of volatility data points within each of the 4 cases versus number of days between option expiry and futures maturity: (i) Light green: *best case* true/true: both CME mid and OTC mid are within the bid-ask spread, (ii) Green: true/false: CME is within and OTC is not, (iii) Blue: false/true: CME is not and OTC is within and (iv) Purple: *worst case* false/false: both CME and OTC are not within the bid-ask spread. The blue line shows the total number of data points used to build each time bucket.

standard assumption made on the interest rate dynamic does not seem to impact the converter performance at longer Γ. Thus, we do not need to use a more sophisticated form of interest rate modeling.

3.4 Comparing CME and The OTC FX Option Markets

The OTC FX option data aggregated by the data provider represents the OTC Dealer-to-Dealers market (D2D). The OTC FX option D2D market allows large dealers, mostly international institutional banks, to trade FX options with each other. Given that dealers are large institutions, we anticipate better pricing when dealers trade with each other in the D2D market compared to when they trade against clients in the Dealer-to-Client (D2C) market. As such, the D2D market is expected to be very efficient: accurate pricing, lower mid-volatility compared to what should be observed in the D2C markets. The CME FX option market is a D2C market: FXO dealers and other market makers provide limit orders on CME options while buy-side clients execute option strategies using aggressive orders.

In this section, we look at empirical evidence to answer a few questions defined in the introduction:

– What evidence can we find on valuation alignment of the two markets? How often does CME mid-volatility fall inside the OTC spread range, and OTC mid-volatility fall inside the CME spread range?

– Is there a valuation bias (i.e. CME being at a higher option valuation) ?
– Statistically, do we see significant cointegration in the time series of OTC and CME FXO demonstrating that simple pairs trading strategies can be executed between the markets?

To answer the first set of questions, we start by comparing CME and OTC mid-volatilities using the measure *alignment of markets* defined in 3.3. Figure 13 presents the percentage of each of the 4 cases for EURUSD and JPYUSD. Overall, we can see that the percentages for each of the four cases are very similar for both currency pairs. Furthermore, both the CME and OTC markets are very close to each other. The *best case* scenario has the largest percentage with about **two-thirds**, at 65.0% and 67.4%, while the *worst case* scenario is the smallest accounting for **one-sixth** of the data, at 15.5% and 13.8%, for USDJPY and EURUSD respectively. Finally, the remaining one-fifth is split between the two middle cases, at 19.6% and 18.9%. The alignment results are very high, demonstrating that the two markets are in fact highly aligned, confirming their strong relationship especially considering their large structural differences.

	TT	TF	FT	FF
USDJPY	65.0%	8.4%	11.2%	15.5%
EURUSD	67.4%	8.5%	10.4%	13.8%

Fig. 13: Summary of the 4 cases for EURUSD and USDJPY: true/true, true/false, false/true and false/false.

Figures 23 and 30 show the four Booleans cases as a function of the option time-to-expiry for EURUSD and USDJPY, respectively. When the time-to-expiry increases, the alignment decreases for EURUSD. Markets' alignment (case: true/true) goes from 70% for short- and mid-term expiries up to 65% for longer expiries, while markets' non-alignment (case false/false) increases from 12% to 15%, when option expiry increases. However, such trend is inverted for USDJPY. Figure 24 and Figure 31 present the percentage of volatility data points within each of the 4 Booleans cases versus options delta for EURUSD and USDJPY, respectively. This time both pairs show similar results, CME and OTC market alignment is higher around the at-the-money strike prices and decreases as we get further away from it. ATM shows 70 to 75% of the volatilities being aligned (case: true/true) versus 60% to 67% for the 10 delta for both currency pairs.

We can conclude and answer the first question: the data shows that two markets are highly aligned for EURUSD and USDJPY, with most cases showing that market-mids are within the bid-ask range of the other market.

We continue by analyzing the percentage of times CME mid-volatility is premium (discount) to the OTC market to answer the second set of questions. On average, across expiry and delta, CME is at discount 46% of the time for EURUSD and 55% for USDJPY. Figures 25 and 32 show the percentage when CME is at a volatility premium or discount to the OTC market as a function of expiry for EURUSD and USDJPY. For EURUSD, CME is at par for short-dated and long-dated options and around 43% discount for mid-term options. For USDJPY, there is a clear linear expiry trend, with the CME being at discount 60% for short-dated, 55% for mid-term, and around 47% for long expiries. Looking at the delta trend for EURUSD in Figure 26, the 25 deltas shows CME mostly at discount (62% for a 25 call) while the at-the-money is almost at par (48%) and the 10 deltas are at premium (around 40%). The delta trend for USDJPY is very similar to EURUSD but shifted upward as showed in Figure 33. Figures 27 and 34 present for both cases, discount and premium, what is the median and standard deviation of the differences. For the two currency pairs, both the median and standard deviation of the differences decrease from 1-day to 3-month as the expiry increases and then goes back up from 3-month to 1-year. However, comparing the level of the mean and standard deviation the largest differences are seen on the 1-day and 1-week expiries. Additionally, the 1-day and 1-week expiries also show the largest difference between premium and discount statistics.

To conclude and answer the second question: CME and OTC are at par from a valuation standpoint for most of the grid points for both currency pairs. The level of premium/discount are different for EURUSD and USDJPY. While CME is at discount 46% of the time for EURUSD, the number increases to 55% for USDJPY. CME seems to be consistently at discount for the 25 calls and puts and at premium for the 10 calls and puts.

Finally, to answer the last set of questions, we look at a cointegration test of the option premium time series. We use the augmented Engle-Granger two-step cointegration test available in the statsmodels[20] package. The null hypothesis is that there is no cointegration, while the alternative hypothesis is that there is cointegrating relationship. If the p-value is small, below a critical size, for example 5%, then we can reject the hypothesis that there is no cointegrating relationship, the p-values above are not cointegrated. Figure 14 shows the p-value results for call and put premium EURUSD time series. Most tests reject the hypothesis that there is no cointegration. Only 6 out of 50 times series have p-values higher than 5%. These exceptional cases are concentrated in 4-month and 5-month expiries and at 10-delta. Figure 15 shows similar results for USDJPY. All tests reject the hypothesis that there is no cointegration except for 3 p-values at the 1-year expiry.

We can now answer the last question: we do see significant cointegration in the time series of OTC versus CME calls and puts option premiums for most of the grid data point.

4 Conclusion

This paper is based on the the previous work of Cinchibuch [7]. We improve the conversion approach by using a weaker assumption than Cinchibuch's.

[20] https://www.statsmodels.org/stable/generated/statsmodels.tsa.stattools.coint.html

Expiry	10C	10P	25C	25P	50C
1D	7.5%	0.0%	0.3%	0.0%	0.0%
1W	0.0%	0.0%	0.0%	0.0%	0.0%
4W	0.0%	0.0%	0.0%	0.0%	0.0%
8W	1.5%	5.6%	1.1%	1.6%	2.6%
12W	1.0%	4.6%	0.1%	2.8%	3.5%
4M	35.2%	11.1%	5.0%	9.1%	3.9%
5M	11.5%	0.5%	0.1%	0.3%	0.2%
6M	0.6%	0.0%	0.0%	0.0%	0.2%
9M	0.0%	0.1%	0.0%	0.2%	1.2%
1Y	0.0%	0.0%	0.0%	0.0%	0.2%

Fig. 14: EURUSD

Expiry	10C	25C	50C	25P	10P
1D	0.2%	0.1%	0.0%	0.1%	0.2%
1W	0.0%	0.6%	0.3%	1.0%	0.0%
4W	0.0%	0.0%	0.0%	0.0%	0.1%
8W	0.0%	0.0%	0.2%	0.0%	0.0%
12W	0.0%	3.0%	0.0%	0.1%	0.9%
4M	0.0%	0.6%	0.0%	0.0%	0.0%
5M	0.0%	0.0%	0.0%	0.1%	0.0%
6M	0.0%	0.0%	4.3%	0.0%	0.0%
9M	0.0%	2.0%	0.5%	0.0%	0.0%
1Y	87.3%	94.2%	11.9%	0.0%	0.0%

Fig. 15: USDJPY

Fig. 16: The p-value of the cointegration test of the call and put premium time series at different delta and expiry. Values above 5% validate the null hypothesis that there is no cointegration.

Furthermore, we choose the most recent market data with the least amount of products misalignment, due to cut-off or Exercise-style, to compare the two FXO markets, CME and OTC. We demonstrated that the two markets have a strong relationship, they are aligned most of time, have similar valuation across most expiries and deltas, and they show cointegration between them.

The current approach promotes the conversion to go from CME FX option to OTC-equivalent because it is easier to compare the markets on the OTC grid. However, it is clear that using the same conversions detailed on the paper, we can also transform OTC FX option volatilities to CME-equivalent option premium. This reciprocal approach can be used for example by someone interested in trading an OTC FXO position that, instead, execute the similar CME-equivalent strategy on CME Globex.

Further research on the topic could include modeling the interest rate dynamic as stochastic to verify the impact of the assumption or adapting the current SABR model to be tenor-dependent.

A Arbitrage Free Futures Price

In Equation A.1, we re-writes the equations (6) to (9) of Gunther, et al [12] to define the inverse forward futures ratio (IFFR) using our notation. Gunther et al were focused on the S&P 500 futures contract so some adjustments are needed to transfer to the FX market.

$$\begin{aligned} IFFR &= \frac{F_{t,T_1}}{F_{t,T_2}} = \frac{E_t[S(T_1)]}{F_{f,T_2}} \\ &= \frac{E_t[E_{T_1}[S(T_2)D(t,T_1,T_2)]]}{F_{f,T_2}} = \frac{E_t[S(T_2)D(t,T_1,T_2)]}{F_{f,T_2}} \\ &= \frac{E_t[S(T_2)]E_t[D(t,T_1,T_2)]}{F_{f,T_2}} + \frac{\mathrm{cov}_t\left(S(T_2), D(t,T_1,T_2)\right)}{F_{f,T_2}} \\ &= E_t[D(t,T_1,T_2)] + \mathrm{cov}_t\left(\frac{S(T_2)}{E_t[S(T_2)]}, D(t,T_1,T_2)\right) \end{aligned} \tag{A.1}$$

Assuming a non-stochastic interest rate, we can write:

$$IFFR = D(t,T_1,T_2) \tag{A.2}$$

and

$$\begin{aligned} F_{T_1,T_2} &= S_{T_1} IFFR \\ &= S_{T_1} D(t,T_1,T_2) \end{aligned} \tag{A.3}$$

When the interest rate is constant or non-stochastic, the covariance term $\mathrm{cov}_t(.)$ is equal to zero and $E_t[D(t,T_1,T_2)] = D(t,T_1,T_2)$

As a consequence, F_{T_1,T_2} is equal to $S_{T_1} D(t,T_1,T_2)$.

To apply the above to the FX market, we set $E_t[D(t,T_1,T_2)] = E_t[\frac{D^f(t,T_1,T_2)}{D^d(t,T_1,T_2)}]$. This ratio is equivalent to the price of an OTC FX swap between T_1 and T_2 traded at time t.

B Implying Futures Fair Value from Options Premium

For European-style options, given prices for an option contract with expiry T, we use put-call parity to imply the underlying futures rate F_{T_1}. Here T_1 is the expiry of underlying futures contract. The underlying futures F_{T_1} and the discount factor D_T are obtained by solving a linear regression of $C_K - P_K$ on K.

$$C_K - P_K = D_T(F_{T_1} - K) = D_T F_{T_1} - D_T K \tag{B.1}$$

$$F_{T_1} = \frac{D_T F_{T_1}}{D_T} = -\frac{\text{intercept}}{\text{coefficient}} \tag{B.2}$$

If there are multiple option contracts sharing the same underlying futures but that expire on different dates, the implied underlying futures price is obtained by averaging the futures price implied from each option contract.

We use the data of March 16, 2020 as an example. On that date, the price information is available for the three options expiring on January 8, February 5, and March 5, 2021. All those three options share one underlying futures that expires on March 15, 2021 (the fourth quarterly futures contract) whose price is not observed. We imply the underlying futures price F with the following steps. First, we run a regression on each of the three options and calculate the implied futures price. Figures 17, 18, and 19 show the regression scatter plots. The approach works very well on options data with all options at different strike prices aligning almost perfectly with the regression line showing that the CME FX option market respects the put-call parity no arbitrage constraint. Figure 22 shows the root-mean-squared-error (RMSE) and the implied underlying future price. Second, we take the average of the

3 implied future prices to get the implied futures of 1.12458071. Figure 20 shows the result from the regression approach. We can see from the plot that the level of the implied futures price is reasonable.

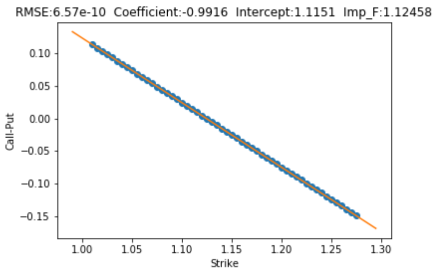

Fig. 17: January 8, 2021.

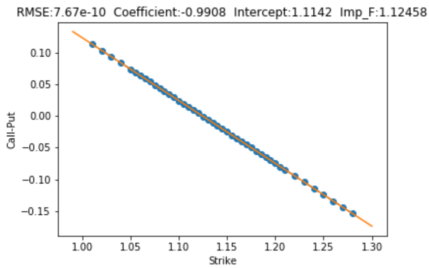

Fig. 18: February 5, 2021.

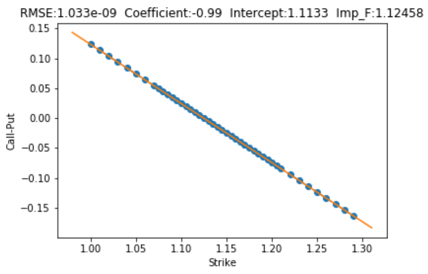

Fig. 19: March 5, 2021.

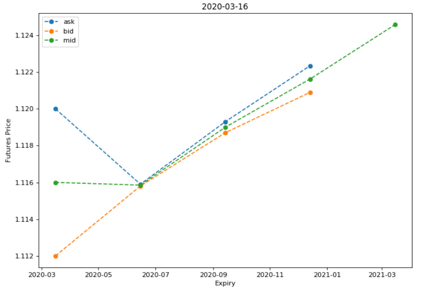

Fig. 20: Futures curve, March 16, 2020

Fig. 21: EURUSD regression plots for the option expiries January 8, February 5, and March 5, 2021 and resulting futures curve for March 16, 2020. The first futures contract is maturing on that date which explains the large bid-ask spread (most market participants roll their future position prior to maturity). The March 2021 futures is the result of the regression / averaging approach.

Option expiry T	*Implied F*	*RMSE*
1/8/2021	1.124580269	6.57E-10
2/5/2021	1.124579056	7.67E-10
3/5/2021	1.124582806	1.03E-09

Fig. 22: Summary of implied futures regression statistics for the March 2021 futures contract.

C Smile Interpolation: SABR model

A volatility model allowing moneyness interpolation is needed due to OTC delta and strike price listing convention differences. We calibrate the Stochastic Alpha Beta Rho (SABR) model of Hagan, et al. [13] to the CME OTC-equivalent option premium at each listed expiry date. We transform the original SABR parameters ρ, ν and make them orthogonal to each other by defining their Cartesian equivalent ν_1, ν_2 with $\nu_1 = \rho\nu$ and $\nu_2 = \sqrt{\nu_2^2 + \nu_1^2}$.

We calibrate SABR parameters on each mid, bid and ask volatility smiles. Calibrating volatility surfaces is an optimization problem. We use the Levenberg-Marquardt optimizer to minimize the root mean square error (RMSE) between the observed OTC-equivalent option premium, V_i, and SABR option premium, $V(K_i, T, \sigma_{SABR}(\alpha, \nu_1, \nu_2))$ for each strike i. The objective function is defined below in Equation C.1.

$$\begin{aligned} \underset{\alpha,\nu_1,\nu_2}{\text{minimize}} \quad & \sum_{i=1}^{N} \omega_i (V_i - V(K_i, T_1, \sigma_{SABR}(F, K_i, \alpha, \beta, \nu_1, \nu_2)))^2 \\ \text{subject to} \quad & 0 < \alpha < 1 \\ & -\infty < \nu_1 < \infty \\ & 0 < \nu_2 < \infty. \end{aligned} \tag{C.1}$$

Where T_1 is the option time-to-expiry in years, N is the number of option contracts with time-to-expiry T_1, ω_i is a weighted function, V_i is the observed OTC-equivalent option premium for option i with K_i strike price. The SABR model has four parameters: $\{\alpha, \beta = 1, \nu_1, \nu_2\}$. For FX, the market convention to assume $\beta = 1$, also called the *FX-SABR model*. ν_1 and ν_2 are the Cartesian orthogonal transformation of the original SABR parameters ρ and ν as defined above. The SABR volatility (σ_{SABR}) is defined by the SABR four parameters, underlying instrument F, strike price K and time-to-expiry T_1, as shown in Equation C.2:

$$\begin{aligned} \sigma_{\text{SABR}} &= \frac{\text{nominator}}{\text{denominator}} \frac{z}{X(z)} \\ \nu &= \sqrt{\nu_1^2 + \nu_2^2} \\ \rho &= \frac{\nu_1}{\nu} \\ \text{nominator} &= \alpha \left(1 + \left(\frac{(1-\beta)^2 \alpha^2}{24(FK)^{1-\beta}} + \frac{\rho\beta\nu\alpha}{4(FK)^{\frac{1-\beta}{2}})} + \frac{(2-3\rho^2)\nu^2}{24} \right) T_{exp} \right) \\ \text{denominator} &= (FK)^{\frac{(1-\beta)}{2}} \left(1 + \frac{(1-\beta)^2 log(\frac{F}{K})^2}{24} + \frac{(1-\beta)^4 log(\frac{F}{K})^4}{1920} \right) \\ z &= \frac{\nu}{\alpha} \cdot (F \cdot K)^{\frac{(1-\beta)}{2}} log(F/K) \end{aligned} \tag{C.2}$$

We want the optimizer to solve for an unconstrained problem, however, as defined in Equation C.1 we have upper and lower boundaries for the different parameters. To go from a constrained optimization problem to an unconstrained optimization problem, we map the results from the optimizer to bounded intervals. The function to transform $x \in (-\infty, \infty)$ to $x' \in (L, U)$ is defined in Equation C.3:

$$x' = \frac{1}{2} ((U + L) + (U - L) \cdot \tanh(\gamma x)) \tag{C.3}$$

Where γ is a nonlinear scaling parameter, U is the upper bound, L is the lower bound and tanh(.) is the hyperbolic tangent function. Theoretically, SABR parameters can be unbounded on either the upper, lower, or both sides. For example, α is bounded on the upper side and ν_1 is unbounded on both sides. However, we set constraints on parameters boundaries to make sure the parameter values are reasonable. Once SABR parameters are solved for every observed expiry, it is easy to get implied volatility for any strike. However, the OTC grid requires finding delta, not strike, so we need a delta-to-strike transformation approach.

D Delta to Strike Transformation

The strike price from delta is calculated when building standard OTC implied volatility surface. The FXO grid is defined in delta terms and because SABR is defined in strikes we need to convert the standard grid delta into strikes. Equation D.1 defines pip delta: $\Delta_{\text{pips,spot}}$. The option strike can be backed out from the equation using an optimizer, for example, the Brent–Dekker method.

$$\begin{aligned} \Delta_{\text{pips,spot}} &= \omega \cdot e^{-r_f \tau} \cdot \Phi(\omega d_1) \\ d_1 &= \frac{log(\frac{F}{K})}{\sigma_{\text{SABR}}\sqrt{T}} + \frac{1}{2}\sigma_{\text{SABR}}\sqrt{T} \end{aligned} \tag{D.1}$$

where:

- Φ is the standard normal cumulative distribution function.
- $\omega = 1$ for call and -1 for put.
- r_f = foreign (base) currency simple interest rate
- σ_{SABR} implied volatility from calibrated SABR volatility
- F option underlying

As the optimizer may give us more than one solution, we put the following constrains on strike K as define in D.2 :

$$\begin{aligned} K_{\text{lower bound}} &= F \cdot e^{-n\alpha\sqrt{T}} \\ K_{\text{upper bound}} &= F \cdot e^{+n\alpha\sqrt{T}} \end{aligned} \tag{D.2}$$

n represents number of standard deviations. For example, the value can be set to 5.

E Figures for EURUSD and USDJPY

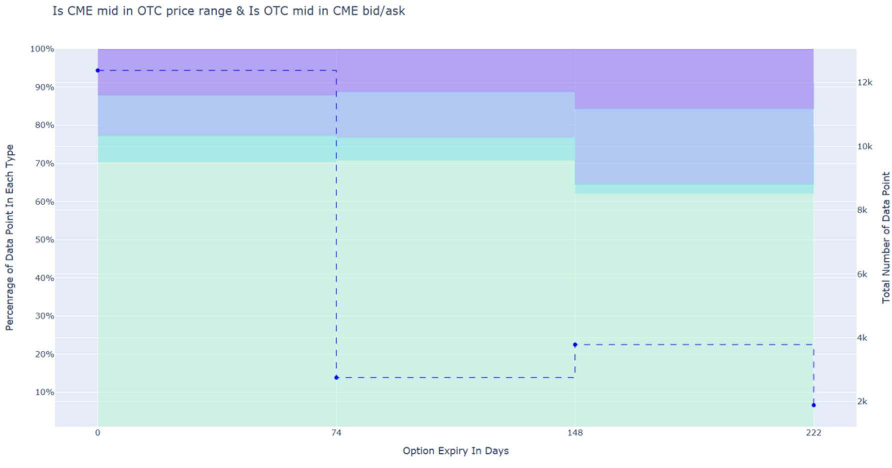

Fig. 23: EURUSD - Percentage of volatility data points within each of the 4 cases versus option expiry: (i) Light green: True, True: both CME mid and OTC mid are within the bid-ask spread, (ii) Green: True, False: CME is within and OTC is not, (iii) Blue: False, True: CME is not and OTC is within, and (iv) Purple: False, False: both CME and OTC are not within the bid-ask spread. The blue line shows the total number of data points used to build each expiry bucket. We divided the data into 4 buckets {0-74}, {74-148}, {148-222}, and {222-365}.

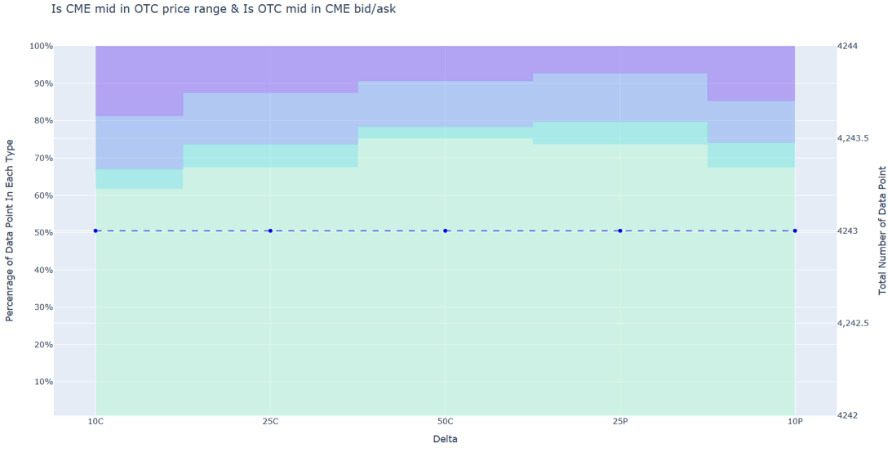

Fig. 24: EURUSD - Percentage of volatility data points within each of the 4 cases versus option delta: (i) Light green: True, True: both CME mid and OTC mid are within the bid-ask spread, (ii) Green: True, False: CME is within and OTC is not, (iii) Blue: False, True: CME is not and OTC is within, and (iv) Purple: False, False: both CME and OTC are not within the bid-ask spread. The blue line shows the total number of data points used to build each delta bucket.

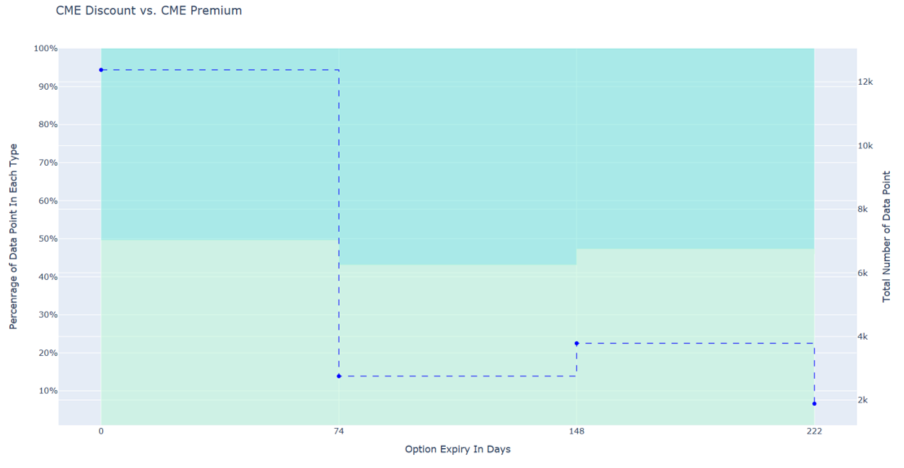

Fig. 25: EURUSD - Percentage of CME volatility data points at premium (dark green) or at discount (light green) to OTC versus expiry. CME premium indicates CME mid volatility is superior to OTC mid volatility. The blue line represents the number of data points.

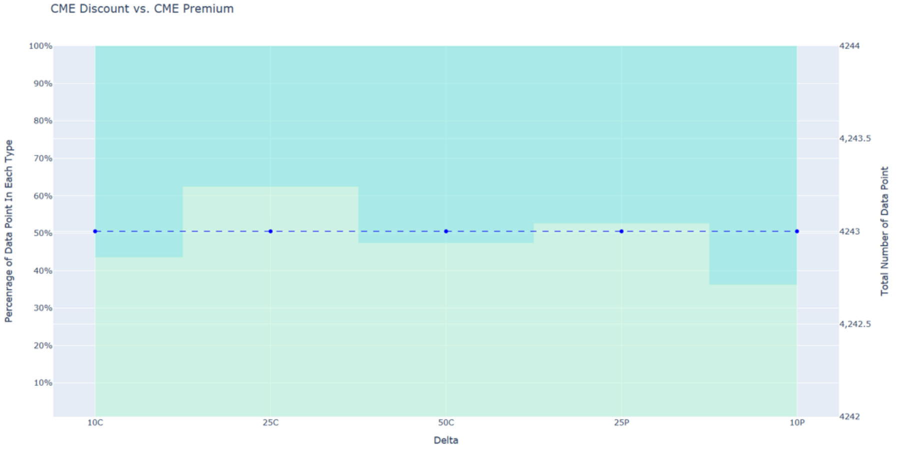

Fig. 26: EURUSD - Percentage of CME volatility data points at premium (dark green) or at discount (light green) to OTC versus delta. CME premium indicates CME mid volatility is superior to OTC mid volatility. The blue line represents the number of data points.

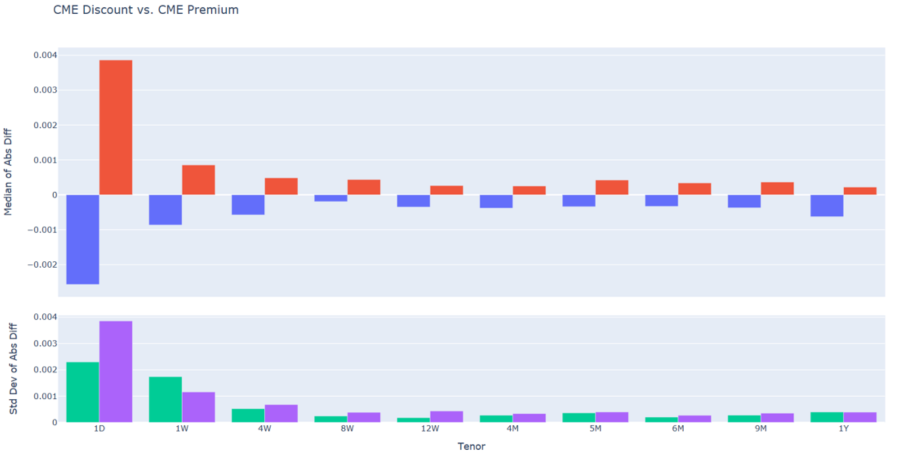

Fig. 27: EURUSD - Comparison of median (top) and standard deviation (below) of the difference in volatility between OTC mid and CME mid by option expiry. The median and standard deviation is, respectively, blue and green when CME is at discount to OTC mid-vol, and red and purple when CME is at premium.

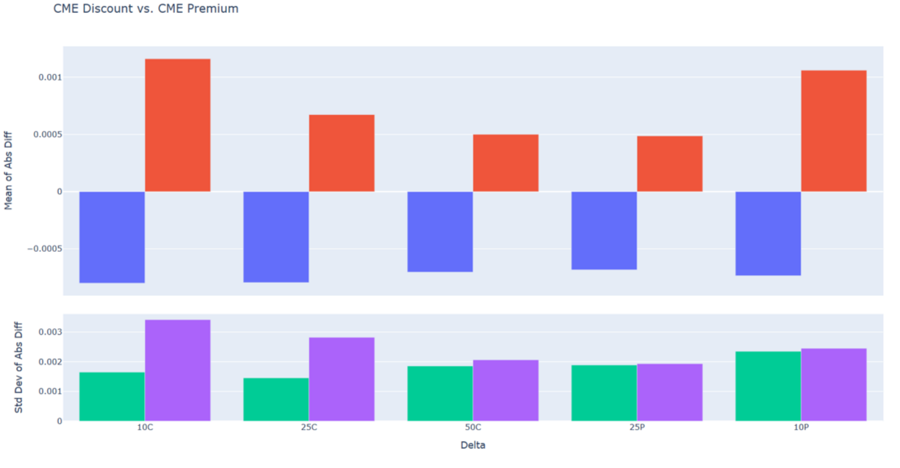

Fig. 28: EURUSD - Comparison of median (top) and standard deviation (below) of the difference in volatility between OTC mid and CME mid by delta. The median and standard deviation is, respectively, blue and green when CME is at discount to OTC mid-vol, and red and purple when CME is at premium.

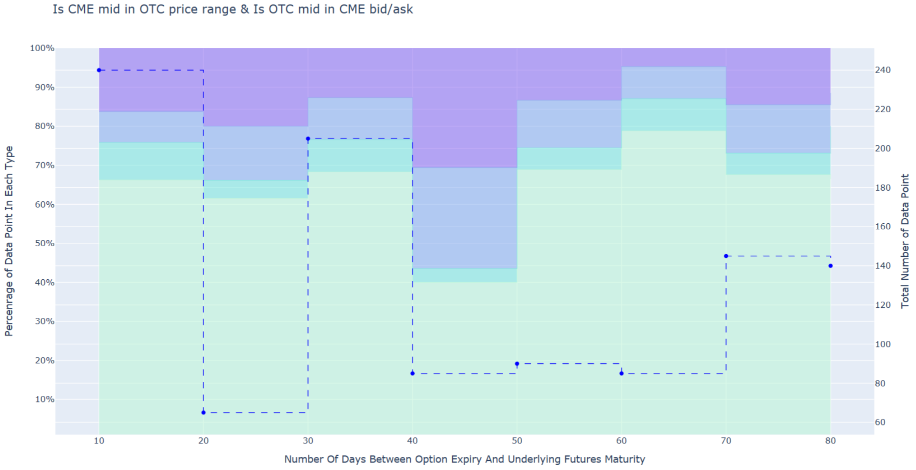

Fig. 29: USDJPY - Percentage of volatility data points within each of the 4 cases versus number of days between option expiry and futures maturity: (i) Light green: True, True: both CME mid and OTC mid are within the bid-ask spread, (ii) Green: True, False: CME is within, and OTC is not, (iii) Blue: False, True: CME is not and OTC is within, and (iv) Purple: False, False: both CME and OTC are not within the bid-ask spread. The blue line shows the total number of data points used to build each time bucket.

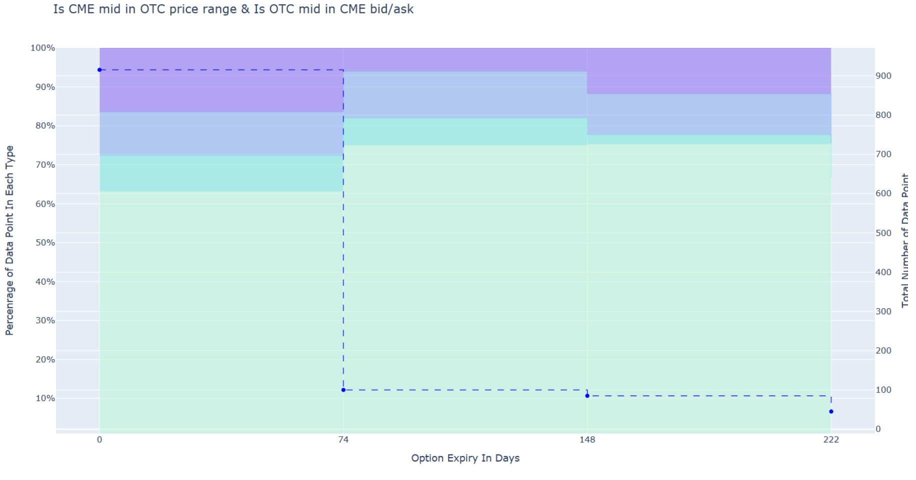

Fig. 30: USDJPY - Percentage of volatility data points within each of the 4 cases versus option expiry: (i) Light green: True, True: both CME mid and OTC mid are within the bid-ask spread, (ii) Green: True, False: CME is within and OTC is not, (iii) Blue: False, True: CME is not and OTC is within, and (iv) Purple: False, False: both CME and OTC are not within the bid-ask spread. The blue line shows the total number of data points used to build each expiry bucket. We divided the data into 4 buckets {0-74}, {74-148}, {148-222} and {222-365}.

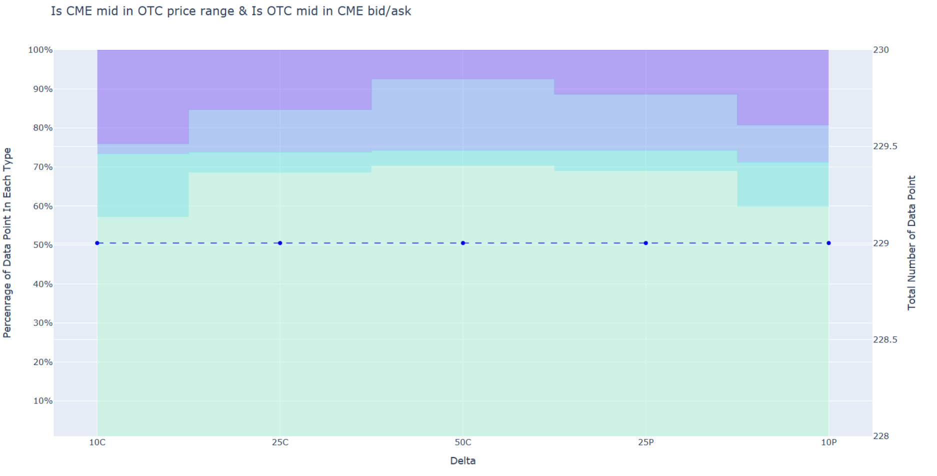

Fig. 31: USDJPY - Percentage of volatility data points within each of the 4 cases versus option delta: (i) Light green: True, True: both CME mid and OTC mid are within the bid-ask spread, (ii) Green: True, False: CME is within and OTC is not, (iii) Blue: False, True: CME is not and OTC is within, and (iv) Purple: False, False: both CME and OTC are not within the bid-ask spread. The blue line shows the total number of data points used to build each delta bucket.

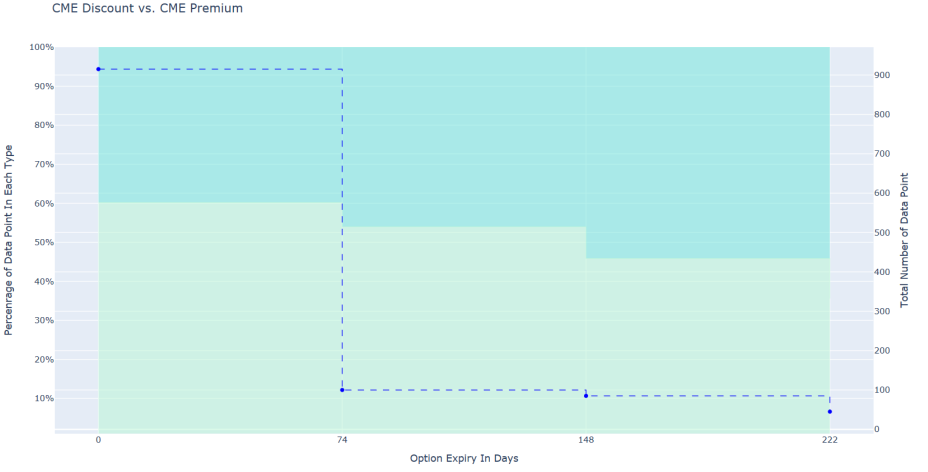

Fig. 32: USDJPY - Percentage of CME volatility data points at premium (dark green) or at discount (light green) to OTC versus expiry. CME premium indicates CME mid volatility is superior to OTC mid volatility. The blue line represents the number of data points.

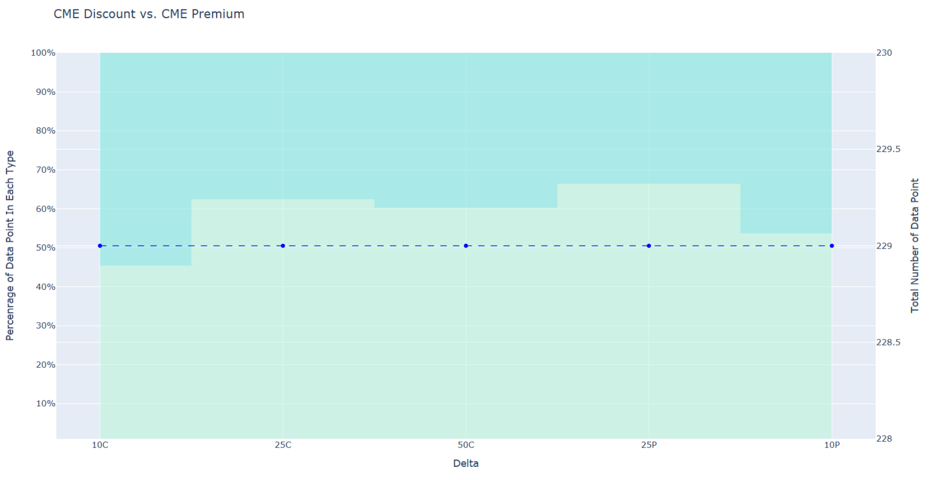

Fig. 33: USDJPY - Percentage of CME volatility data points at premium (dark green) or at discount (light green) to OTC versus delta. CME premium indicates CME mid volatility is superior to OTC mid volatility. The blue line represents the number of data points.

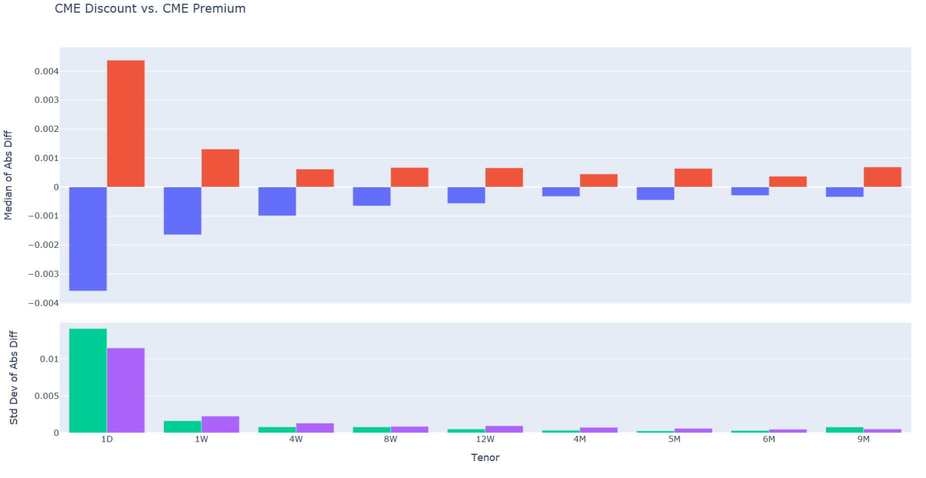

Fig. 34: USDJPY - Comparison of median (top) and standard deviation (below) of the difference in volatility between OTC mid and CME mid by option expiry. The median and standard deviation are, respectively, blue and green when CME is at discount to OTC mid-vol, and red and purple, when CME is at premium.

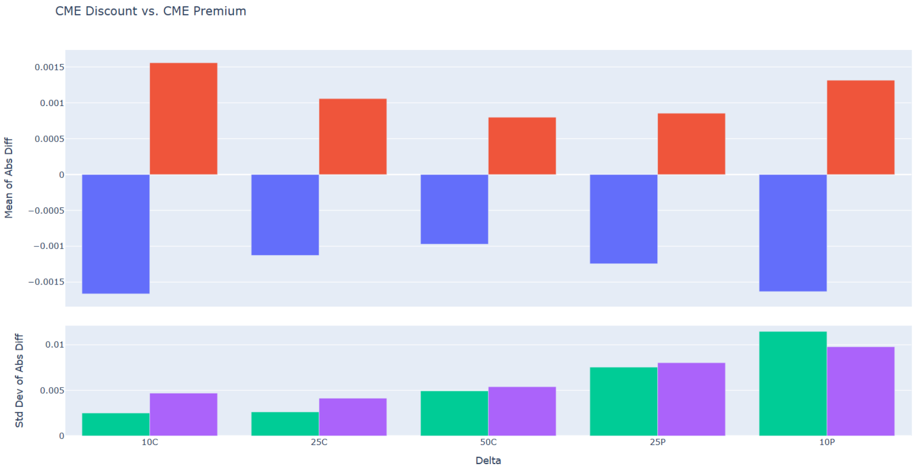

Fig. 35: USDJPY - Comparison of median (top) and standard deviation (below) of the difference in volatility between OTC mid and CME mid by delta. The median and standard deviation are, respectively, blue and green when CME is at discount to OTC mid-vol, and red and purple, when CME is at premium.

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