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DIGITAL ASSETS UNLEASHED

CME CF Bitcoin Real Time Index (BRTI)

Methodology Guide

Version: 2

Version Date: 06 March 2017

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1 Version History

Version	Version Date	Changes to Previous
1	10 November 2016	N/A
2	06 March 2017	Added description of treatment of erroneous data. Changed calculation methodology from static to dynamic depth. **Effective: 18 March 2017, 16:30:00 London time.**

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2 Definitions

API: Application programming interface.

Calculation Time: Any time as of which the BRTI is published.

Constituent Exchange: A bitcoin trading venue elected by the Bitcoin Pricing Product Oversight Committee to serve as pricing source for the calculation of the BRTI.[1]

Relevant Order Book: The universe of the currently unmatched limit orders to buy or sell bitcoin versus U.S. Dollars on a Constituent Exchange that is reported through its API to the Calculation Agent.

Retrieval Time: The time, as given by the server clock of the Calculation Agent, as of which the Relevant Order Book of a Constituent Exchange is requested by the Calculation Agent through the API of the Constituent Exchange.

[1] Constituent Exchanges are published on the Administrator's website.

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3 Summary Description

The CME CF Bitcoin Real Time Index ("BRTI") is a real time index of the U.S. Dollar price of one bitcoin. It is representative of current bids and offers of market participants to buy or sell bitcoin on Constituent Exchanges. It is geared towards timeliness and represents an unbiased estimator of the instantaneous bitcoin price.

Name	CME CF Bitcoin Real Time Index
CME Ticker Symbol	BRTI
Administrator	Crypto Facilities Ltd.
Calculation Agent	Crypto Facilities Ltd.
Description	Instantaneous U.S. Dollar price of one bitcoin
Calculation Methodology	Real time aggregation of order book data of Constituent Exchanges
Dissemination Time	Approximately every second of each day for the entire year including weekends and holidays.
Dissemination Precision	0.01 U.S. Dollars

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4 Methodology and Rules

4.1 Methodology

4.1.1 Qualitative Description

The BRTI is calculated in real time based on the Relevant Order Books of all Constituent Exchanges. An order book is a list of buy and sell orders with associated limit prices and sizes that have not yet been matched due to lack of supply or demand to trade at that price. It therefore informs about the price at which a trader can buy or sell a certain amount of bitcoins as of now. In line with existing bitcoin market practises, the price/sizes tuples of buy orders ("bids") descend by price and the price/size tuples of sell orders ("asks") ascend by price.

Calculation steps are as follows:

1. At the Effective Time, the Relevant Order Book of each Constituent Exchange is added to a joint list of order books.
2. The joint list of order books is aggregated into one consolidated order book. If the size of a bid or ask order exceeds 100 bitcoins, it enters the consolidated order book with a size of 100 bitcoins, the order size cap.
3. The cumulative bid price-volume curve, ask price-volume curve, mid price-volume curve and mid spread-volume curve are calculated from the consolidated order book at integer bitcoin transaction volume granularity.
    a. The bid price-volume curve maps bitcoin transaction volume to the marginal U.S. Dollar price per bitcoin a seller is required to accept in order to sell this volume to the consolidated order book.
    b. The ask price-volume curve maps a bitcoin transaction volume to the marginal U.S. Dollar price per bitcoin a buyer is required to pay in order to purchase this volume from the consolidated order book.
    c. The mid price-volume curve represents the average of the bid price-volume curve and the ask price-volume curve.
    d. The mid spread-volume curve represents the percentage deviation of the ask price-volume curve from the mid price-volume curve.
4. The utilized depth is calculated as the maximum cumulative volume for which the mid spread-volume curve does not exceed 0.5%. If this volume is less than one, utilized depth is set to one.

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5. The mid price-volume curve is weighted by the normalized probability density of the exponential distribution up to the utilized depth.
6. The BRTI is then given by the sum of the weighted mid price-volume curve obtained in the previous step.

4.1.2 Mathematical Representation

The following table shows the symbols used in the mathematical representation of the BRTI.

Symbol	Name	Description	Type
T	Effective time	The time at which the BRTI is calculated	Parameter
C	Order size cap	The size above which any excess size of a bid or ask order is discarded	Parameter, set to $C = 100$
D	Deviation from mid	The maximum percentage deviation of a limit order from the mid price-volume curve, until which that limit order is used for the calculation of the BRTI	Parameter, set to $D = 0.5\%$
λ	Lambda	A parameter that determines the shape of the probability density function of the exponential distribution	Parameter, set to $\lambda = \frac{1}{0.3\bar{v}_T}$
v	Volume	The independent variable of a price-volume curve	Internal variable
A_T	Ask orders	The ask orders of the consolidated order book as of the effective time, ordered ascending by price	Input
$a_{T,i}$ with $a_{T,i} = (ap_{T,i}, a\dot{s}_{T,i})$, $a_{T,i} \in A_T, a\dot{s}_{T,i} = \min\{as_{T,i}, C\}$	Ask order	The ith price/size ask order pair of the consolidated order book	Input
B_T	Bid orders	The bid orders of the consolidated order book as of the effective time, ordered descending by price	Input

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$\boldsymbol{b_{T,i}}$ with $\boldsymbol{b_{T,i}} = (\boldsymbol{bp_{T,i}}, \boldsymbol{b\dot{s}_{T,i}})$, $\boldsymbol{b_{T,i}} \boldsymbol{\epsilon} \boldsymbol{B}, \boldsymbol{b\dot{s}_{T,i}} = \boldsymbol{\min}\{\boldsymbol{bs_{T,i}}, \boldsymbol{C}\}$	Bid order	The ith price/size bid order pair of the consolidated order book	Input
$\boldsymbol{NF}$	Normalization factor	A parameter chosen such that $\frac{1}{NF}\sum_{v=1}^{\bar{v}_T} \lambda e^{-\lambda v} = 1$	Output
$\boldsymbol{BRTI_T}$	BRTI	The BRTI at time T	Output

Using the above notation, we define the ask price-volume curve, $askPV_T$, the bid price-volume curve, $bidPV_T$, the mid price-volume curve, $midPV_T$, and the mid spread-volume curve, $midSV_T$, in each case as of the effective time T, as:

$$\boldsymbol{askPV_T(v) = ap_{T,j+1}\ where \sum_{i=1}^{j} a\dot{s}_{T,i} < v\ and \sum_{i=1}^{j+1} a\dot{s}_{T,i} \geq v}$$	**Eq. 3a**
$$\boldsymbol{bidPV_T(v) = bp_{T,j+1}\ where \sum_{i=1}^{j} b\dot{s}_{T,i} < v\ and \sum_{i=1}^{j+1} b\dot{s}_{T,i} \geq v}$$	**Eq. 3b**
$$\boldsymbol{midPV_T(v) = \frac{askPV_T(v) + bidPV_T(v)}{2}}$$	**Eq. 3c**
$$\boldsymbol{midSV_T(v) = \frac{askPV_T(v)}{midPV_T(v)} - 1}$$	**Eq. 3d**

The utilized depth, $\bar{v}_{,T}$, is calculated as:

$$\boldsymbol{\bar{v}_{,T} = max(v_i\ where\ midSV_T(v_i) \leq D\ and\ midSV_T(v_{i+1}) > D, 1)}$$	**Eq. 4**

The BRTI as of the effective time T, $BRTI_T$, is then given by:

$$\boldsymbol{BRTI_T = \sum_{v=1}^{\bar{v}_{,T}} midPV_T(v) \frac{1}{NF} \lambda e^{-\lambda v}}$$	**Eq. 5**

4.2 A Note on Properties

The BRTI reflects the instantaneous supply and demand of bitcoin and results in a robust, yet highly timely indication of the current bitcoin price. This is achieved through the following design choices:

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Order Book Data

The BRTI is calculated from order book data, as opposed to, for instance, trade data. Order book data is composed of unmatched limit orders to buy or sell bitcoin. It informs about the price at which a trader can buy or sell bitcoins now or in the future and is therefore forward-looking by nature. Further, absent retrieval constraints, order book data is always up to date. This is in contrast to trade data, which is produced in stochastic intervals only and informs about the price at which bitcoin has traded in the past.

Mid Price Volume Curve

The mid price volume curve represents the average of the marginal price at which a certain amount of bitcoins can be sold and at which that same amount can be bought. By averaging across the mid price volume curve, the BRTI represents a blend of such (hypothetical) transactions at various transaction sizes.

Utilized Depth

The BRTI is calculated from the section of the mid price volume curve for which ask limit orders at a certain depth diverge by no more than 0.5% from the mid price at that depth. It therefore reflects a significant portion of the top of the consolidated order book (as opposed to, for instance, the best bid and ask prices only) but discards limit orders that are less likely to be matched. This makes it a meaningful representation of true bitcoin liquidity and robust to local changes in order books.

Note that utilized depth will always include any crossing section of the consolidated order book, as well as the non-crossing section with ask limit orders up to 0.5% away of the mid price volume curve. If zero size resides in both these sections, utilized depth is set to one. The BRTI is then effectively equal to the mid price of the consolidated order book.

Exponential Weighting

The mid price-volume curve is weighted by the normalized probability density of the exponential distribution. The exponential distribution and its first derivative are monotonically decreasing, resulting in a higher emphasis on the initial section of the mid price-volume curve, i.e. bid and ask prices that are closest to the global best bid and ask price.

Markov Process and Martingale

Indices are frequently based on historical data, such as recent trade prices or volumes. This can result in certain predictability, for instance due to the expectation that stale prices will be updated or that a price will drop out of a weighting window.

By relying solely on order book data, the BRTI is both a Markov process and a martingale. A Markov process is a stochastic process with a probability distribution

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that depends only on the current information set, not on historical information sets. The martingale property implies that the best prediction of the next BRTI value is its current value:

$$E(BRTI_{t+1}|BRTI_1, BRTI_2, \dots, BRTI_t) = BRTI_t$$

This makes the BRTI useful for applications that require an unbiased estimator of the future bitcoin price.

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5 Contingency Calculation Rules

5.1 Delayed Data

Delayed data is treated according to the following rules:

1. If the Retrieval Time of the Relevant Order Book of a Constituent Exchange is at least 30 seconds older than the Calculation Time, the Constituent Exchange is disregarded in the calculation of the BRTI for that Calculation Time.

If the Retrieval Times of the Relevant Order Books of all Constituent Exchanges are each at least 30 seconds older than the Calculation Time, a BRTI calculation failure occurs for that Calculation Time (see Section 5.4).

5.2 Erroneous Data

All Relevant Order Books are subject to an automated screening for erroneous data according to the following rules:

1. If a Relevant Order Book contains at least one instance of a non-numeric or non-positive limit price or size, it is flagged as erroneous.

2. If the format of a Relevant Order Book deviates from the expected format such that it cannot be parsed, it is flagged as erroneous.

3. If the Relevant Order Book contains zero bid orders or zero ask orders, it is flagged as erroneous.

Relevant Order Books flagged as erroneous for a given Calculation Time are disregarded in the calculation of the BRTI for that Calculation Time.

If the Relevant Order Books of all Constituent Exchanges are flagged as erroneous for a given Calculation Time, a BRTI calculation failure occurs for that Calculation Time (see Section 5.4).

5.3 Potentially Erroneous Data

All Relevant Order Books are subject to an automated screening for potentially erroneous data according to the following rules:

1. For each Constituent Exchange individually, the current mid price is calculated as the average of the highest bid price and the lowest ask price of the Relevant Order Book.

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2. For each Constituent Exchange, the absolute percentage deviation of the mid price, as calculated in the previous step, from the median of the mid prices of all Constituent Exchanges is calculated.

3. If for any Constituent Exchange the absolute percentage deviation, as calculated in the previous step, exceeds 25%, the Relevant Order Book of that Constituent Exchange is flagged as potentially erroneous.

Relevant Order Books flagged as potentially erroneous for a given Calculation Time are disregarded in the calculation of the BRTI for that Calculation Time. The occurrence of any such flag is reported to the Oversight Committee.

If the Relevant Order Books of all Constituent Exchanges are flagged as potentially erroneous for a given Calculation Time, a BRTI calculation failure occurs for that Calculation Time (see Section 5.4).

5.4 BRTI Calculation Failure

If the BRTI cannot be calculated for a given Calculation Time, for instance because

- the Retrieval Times of the Relevant Order Books of all Constituent Exchanges are each at least 30 seconds older than the Calculation Time, or

- all Relevant Order Books are flagged as erroneous or potentially erroneous (see Section 5.2); or

- any other reason or circumstance that prevents the orderly calculation of the BRTI,

then the BRTI for that Calculation Time is not published. The occurrence of any BRTI calculation failure is reported to the Oversight Committee.

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