Zero Rate Delta Ladder Specification

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Document Overview:

With this document, users will be able to populate the delta ladder file that is a required upload for CME CORE and CME Optimizer. This document outlines a method of calculating DV01 for each tenor bucket that allows users to achieve increased margin calculation precision when applying a sensitivity based margin calculation to the delta ladder. The overall amount of variance between a delta ladder based sensitivity calculations using this methodology compared to a full trade by trade revaluation is generally less than 5%. Please note that you are welcome to reach out to cme.core@cmegroup.com with any questions about the contents of this document.

Delta Ladder Description

Overview

A Delta Ladder is the change of an interest rate swap portfolio value given a 1 basis point (0.01%) change to the underlying. We compute zero rate based delta ladders for the purpose of estimating margins.

The number of delta ladders depends on the number of underlying curves that are used to price the portfolio. Each delta ladder contains multiple sensitivities $D = \{d_1, d_2, \dots, d_n\}$, each of the sensitivities d_i correspond to a key tenor τ_i. The sensitivity d_i is defined as the change of the swap portfolio NPV reacting to a 1bp shift at key tenor τ_i on an interest rate zero curve. The sensitivity is defined in the unit of 1bp.

Let R be a zero curve with tenors $T = \{\tau_1, \tau_2, \dots \tau_n\}$, we can formally define the delta ladder as

$$D = \left\{ \frac{\partial NPV}{\partial R(\tau_1)}, \frac{\partial NPV}{\partial R(\tau_2)}, \dots, \frac{\partial NPV}{\partial R(\tau_n)} \right\}$$

$$\text{where} \quad d_i = \frac{\partial NPV}{\partial R(\tau_i)}$$

To numerically approximate the partial derivative calculation, we use a common discretization approach. For each sensitivity component, we have

$$d_i \doteq \frac{\Delta NPV}{\Delta R(\tau_i)} = \left(\frac{NPV_u - NPV_d}{2\ bp} \right)$$

NPV_u is the present value of the portfolio given 1bp up shift of the curve at tenor τ_i; NPV_d is the present value of the portfolio given 1bp down shift of curve at tenor τ_i .

Application in Margin Calculation

There are many ways to compute a delta ladder. The delta ladders described in this document are computed based on interest rate zero curves. Compared to the other methodologies, zero rate delta ladders most accurately approximate margins. This is because the Historical VaR model generates zero rate based shocks.

Delta Ladder Calculation

Input Interest Rate Curves

There is one delta ladder per curve. The curves correspond to the curves that IM methodology uses. As of December 2012, the list of all curves being used in production is

Currency	Index	Curve Name
AUD	6M	AUD_BBSW_6M_ERS
CAD	3M	CAD_BA_3M_ERS
CHF	6M	CHF_LIBOR_6M_ERS
EUR	OIS	EUR_EONIA_1D_ERS
	1M	EUR_EURIBOR_1M_ERS
	3M	EUR_EURIBOR_3M_ERS
	6M	EUR_EURIBOR_6M_ERS
GBP	6M	GBP_LIBOR_6M_ERS
JPY	6M	JPY_LIBOR_6M_ERS
USD	OIS	USD_FEDFUNDS_1D_ERS
	1M	USD_LIBOR_1M_ERS
	3M	USD_LIBOR_3M_ERS
	6M	USD_LIBOR_6M_ERS

Table 1: production zero curves

When computing a Delta Ladder to approximate a margin at date-*t*, we use date-*t* base curves. Each curve is represented as discount factors[1] shown in the following table. The header "#D" is the tenor offset in unit of days. There are 23 input tenors (same as the key tenors used in initial margin calculation):

{ 91D 183D 274D 365D 457D 548D 639D 731D
1096D 1461D 1826D 2192D 2557D 2922D 3287D 3653D
4383D 5479D 7305D 9131D 10958D 14610D 18263D }

Curve Name	91D	…	10958D	14610D	18263D
AUD_BBSW_6M_ERS	0.9912015229		0.3277455222		
CAD_BA_3M_ERS	0.9967518211		0.4546532997	0.3496186751	0.2688299546
…		…			
USD_LIBOR_6M_ERS	0.9986266069		0.4528718212	0.3495639722	0.2783951488
USD_FEDFUNDS_1D_ERS	0.9995558815		0.4976525262	0.3922153983	0.3181367370

[1] Reference sample file: Base_Curves_<yyyymmdd>.csv

Table 2: Discount Factor of each curve
All numbers are for illustration purposes only.

Calculate Delta Ladder

Given an interpolation method, we compute 23 delta values d_i for each Delta Ladder. For a given swap portfolio, the steps to compute Delta Ladders are

(1) Define the underlying curves involved in the portfolio NPV calculation:
For instance assuming a portfolio includes trades priced off USD3M, USD6M and GBP6M index, the NPV of the portfolio calculation involves the following curves USD_LIBOR_3M, USD_LIBOR_6M, GBP_LIBOR_6M and discounting curve USD_FEDFUNDS. Note that for USD and EUR trades, we adopt dual curve margining methodology, hence discounting curve should be included for these two currencies[2].

(2) For a given tenor τ_i, shift up the tenor and compute NPV up-shift:
For example starting from curve USD_LIBOR_3M, shift up one rate at τ_i by 1bp, conduct interpolation on the shifted curve, and compute the NPV based on this shifted curve keeping all other curves unchanged[3]

$$R^u_{USD3M}(\tau_i) = R_{USD3M}(\tau_i) + 1bp$$
$$NPV^u_{USD3M}(\tau_i) = f(R^u_{USD3M}, R_{USD6M}, R_{USDOIS}, R_{GBP6M})$$

(3) For the same tenor, shift down the same curve and compute NPV down-shift: shift down one rate at τ_i by 1bp, conduct interpolation on the shifted curve, and compute the NPV based on this shifted curve keeping all other curves unchanged

$$R^d_{USD3M}(\tau_i) = R_{USD3M}(\tau_i) - 1bp$$
$$NPV^d_{USD3M}(\tau_i) = f(R^d_{USD3M}, R_{USD6M}, R_{USDOIS}, R_{GBP6M})$$

(4) Compute Delta d_i of tenor τ_i for the shifted curve

$$d^i_{USD3M} = \frac{NPV^u_{USD3M}(\tau_i) - NPV^d_{USD3M}(\tau_i)}{2}$$

(5) Repeat step (2) through (4) for all 23 tenors. This yields a Delta Ladder for the shifted curve – in our example USD_LIBOR_3M.

$$D = \left\{\frac{NPV^u_{USD3M}(\tau_1) - NPV^d_{USD3M}(\tau_1)}{2}, \ldots, \frac{NPV^u_{USD3M}(\tau_{23}) - NPV^d_{USD3M}(\tau_{23})}{2}\right\}$$

[2] As of 12/2012 dual curve margin is alive only for USD and EUR

[3] assuming NPV for swap portfolio is some function $f(\cdot)$ of the aforementioned curves

$$NPV = f(R_{USD3M}, R_{USD6M}, R_{USDOIS}, R_{GBP6M})$$

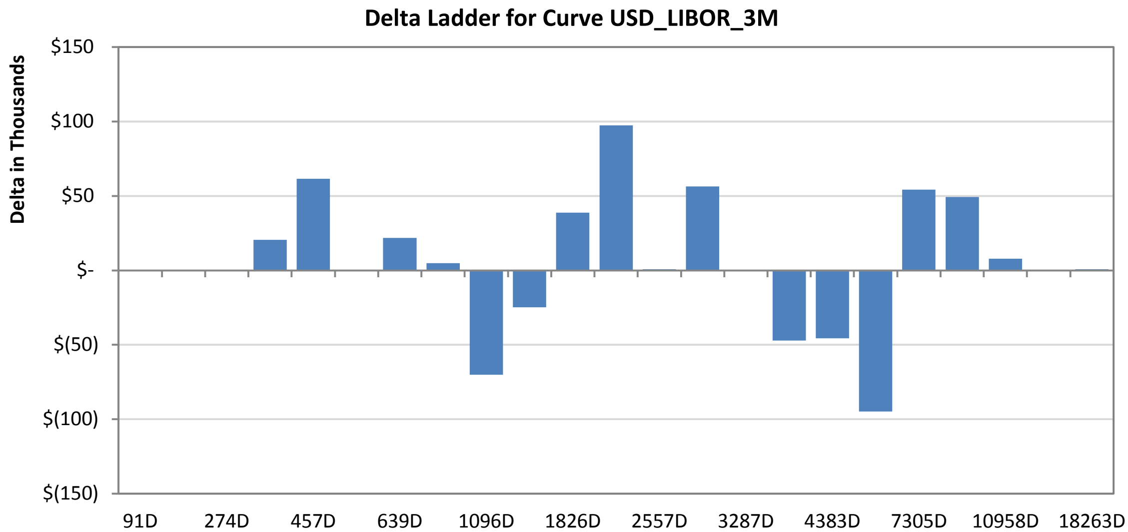

Figure 2: Delta Ladder for One Curve

All numbers are for illustration purposes only.

(6) Repeat step (2) through (5) for all curves, one curve at a time. In this example, we repeat step (2) through (5) for USD_LIBOR_6M, GBP_LIBOR_6M and discounting curve USD_FEDFUNDS. The delta ladder for each curve represents values in local currency as shown in the example table below[4].

Tenor	Delta Ladder		
	USD_LIBOR_3M_ERS	…	GBP_LIBOR_6M_ERS
91D	$ 359.19		£ (119.14)
183D	$ 132.77		£ (110.11)
274D	$ 197.02		£ (103.21)
365D	$ 20,501.34		£ (2,694.72)
457D	$ 61,596.53		£ (12,903.38)
548D	$ 237.03		£ (172.68)
639D	$ 21,739.89		£ (3,182.96)
731D	$ 4,799.43		£ (3,897.44)
1096D	$ (70,174.88)		£ 59,757.03
1461D	$ (24,830.32)		£ 16,061.37
1826D	$ 38,671.56	…	£ (23,010.20)
2192D	$ 97,338.75		£ (32,381.96)
2557D	$ 753.41		£ (239.79)
2922D	$ 56,416.01		£ (17,236.53)
3287D	$ 125.94		£ (88.12)
3653D	$ (47,086.71)		£ 31,830.87

[4] This table is not the final output format of the delta ladder

4383D	$	(45,698.57)		£	23,944.08
5479D	$	(94,826.58)		£	73,004.03
7305D	$	54,132.47		£	(39,522.51)
9131D	$	49,203.20		£	(42,827.96)
10958D	$	7,770.59		£	(5,105.05)
14610D	$	132.54		£	(59.25)
18263D	$	740.33		£	(547.89)

Table 4: Example Delta Ladder for each Currency

All numbers are for illustration purposes only.

Interpolation Example

Log-linear Shift

One approach to interpolate the shifted curves is to use log-linear interpolation on discount factors. In this approach, given a curve R, we first calculate discount factors as:

$$P(0, \tau_i) = \exp(-R_i \cdot \tau_i)$$

for each tenor, τ_i, where R_i is the rate at tenor point τ_i for the curve R. Next, we linearly interpolate the log of the discount factors. The interpolated log discount factors are then converted back to discount factors and can be used to price the swap.

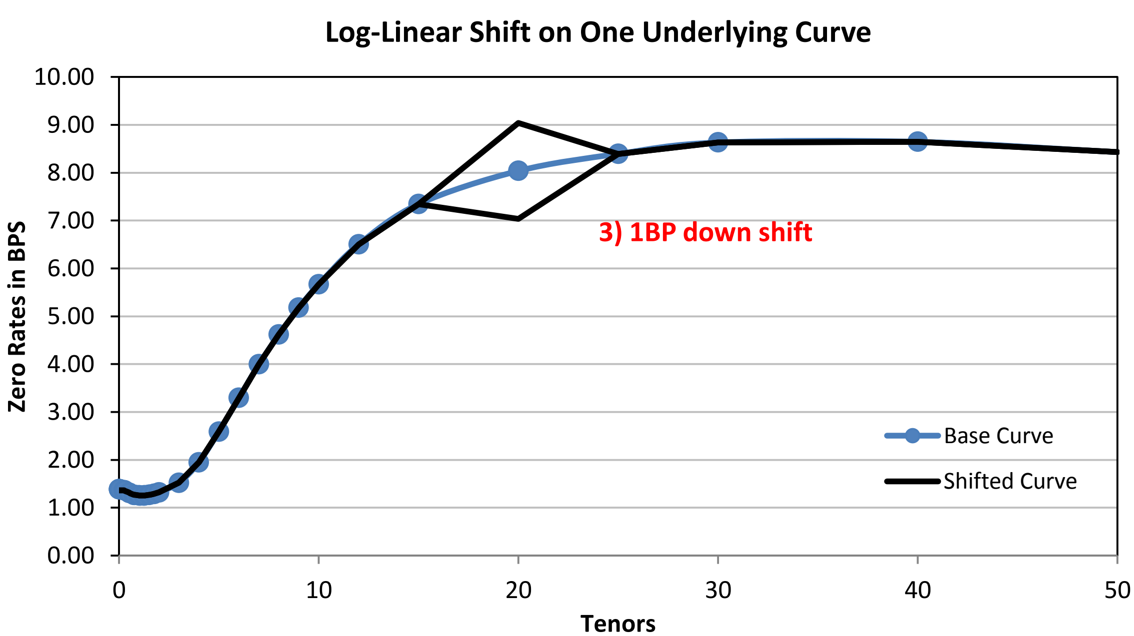

Figure 1: Log-Linear Shift on Zero Curve

All numbers are for illustration purposes only.

Delta Ladder Output

The delta ladder can contain one or more portfolios formatted as below[5]

Tenor Offsets

Portfolio ID	Curve Name	91D	…	10958D	14610D	18263D
A1	AUD_BBSW_6M_ERS	35.7800146	25.7090	64.924348	0	0
A1	CAD_BA_3M_ERS	7.234774	57.76668	82.49858	56.21283	72.14891
A1	…	…	…	…	…	…
A1	USD_LIBOR_6M_ERS	87.9109815	46.0887	23.8254	41.6182	94.7998
A1	USD_OIS_ERS	68.943213	69.279344	81.51971	29.31484	27.029738
BZ350	AUD_BBSW_6M_ERS	56.659142	34.1247	27.439658	0	0
BZ350	…	…	…	…	…	…
BZ350	CHF_LIBOR_6M_ERS	25.02265	16.01724	84.3103	79.748280	95.2147
KING40	AUD_BBSW_6M_ERS	32.555	38.5693	42.29295	0	0
KING40	CAD_BA_3M_ERS	93.617862	38.56905	22.045036	53.621464	90.08169
KING40	CHF_LIBOR_6M_ERS	0.1178	38.5905	23.491108	92.1105	38.06684
…	…	…	…	…	…	…

Delta Ladder of Portfolio KING40
for curve CAD_BA_3M_ERS
denominated in CAD

[5] Reference "delta_ladder_format.csv