Options on Futures on US Treasuries and S&P 500

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Options contracts on the S&P 500 equity index futures, and on the 10-year US Treasury note, are among the most active products traded on the Chicago Mercantile Exchange (CME)—only the Eurodollar complex, and the underlying futures for these options, have higher total volume—and hence among the most active derivative products in the world. Options on the SP500 have been heavily studied; options on Treasuries less so. This note compares the nature of the implied volatility surfaces for these two options products, to see what can be learned about the dynamics of the underlying markets.

We have three main conclusions:

1. Treasury options have much smaller skew than SP500. Figure 2 shows the implied volatility surfaces for two representative dates, and Figure 4 shows a nondimensional measure of skew from late 2012 through early 2015.
2. Treasury option skew has decreased and in fact reversed (Figure 4) in 2014. We believe that this represents the market's fear of an interest rate increase in 2012 and 2013, which abated in 2014, only to return in early 2015.
3. Around the well-publicised Treasury price swings on October 15, 2014, the implied volatility for both SP500 and Treasury options rose, and it began to rise several days before Oct. 15. This indicates that first, the motion was not fundamentally a consequence of the increasingly electronic nature of the market for Treasury notes, bonds, and futures, and second, it was not primarily a Treasury phenomenon but contained a significant equity component as well.

Both options products trade with monthly expirations, although the underlying futures products trade only with quarterly expirations (Figure 1). Thus, for example, the options contracts maturing in July, August, and September all use the September futures contract as underlying. In mid-August the following options contracts will be actively traded: a September options contract on the September futures, as well as October, November, and December options contract on the December futures, and likely several further contracts as well (see the legends in Figure 2). CME makes available at least eight different maturities for Treasury options, and at least seven for the SP500 options.

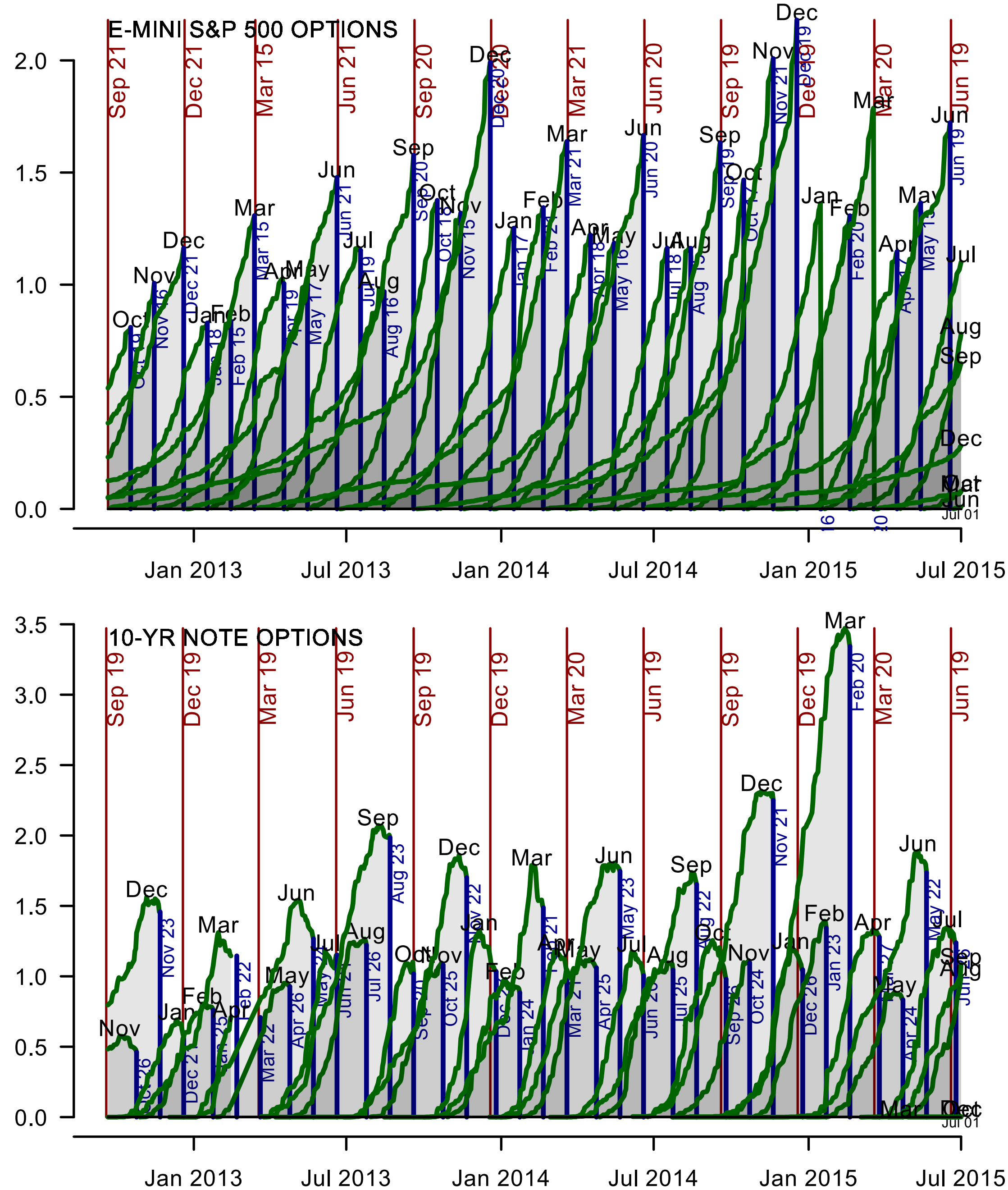

Figure 1: Open interest on options on SP500 index futures and options on 10-year Treasury futures, in millions of lots. Vertical lines show expiration of the underlying futures contracts. SP500 options trade right up to expiration of the underlying, while Treasury options stop trading in the previous month, to avoid the roll period.

We treat all these different futures maturities as equivalent, since the futures prices are very tightly coupled. That is, we will consider all the options contracts active at a particular time to constitute a single set. But we are careful to use the appropriate contract as underlying for each separate option maturity, since there are slight differences between the futures prices.

The expiration of quarterly SP500 options is the same date as the underlying futures contract: trading continues until 8:30 AM Chicago time on the third Friday of each contract month. For quarterly options, this is the same morning that the SP500 index futures price is determined from the opening prices of the underlying stocks, leading to the infamous "triple witching." By contrast, expiration of Treasury options is approximately a week before the end of the month preceding expiration. For quarterly options, this means that the options are expired before the beginning of the roll period, which itself covers a few days before First Notice Date at the beginning of the expiration month. For volatility modeling of both products, it is prudent to exclude the last few trading days before expiration.

These are American options. However, since early expiration is rarely optimal in the current environment of low interest rates, we shall treat them as European.

The underlying Treasury futures contracts themselves have an element of optionality, because of the short position's choice of deliverable asset. In general, this causes the futures price to have less convexity than the price of a similar bond or note. But in a low-rate environment, it would require a large rate change to cause the cheapest to deliver contract to shift from the shortest duration product. Thus we believe this optionality can be neglected for the study of options on futures.

1 Implied Volatility

Figure 2 shows implied volatilities for the entire set of strikes and expirations on one date for each product. In these graphs, the vertical axis is $v = \sigma_{\mathrm{BS}}^2$. The horizontal axis is scaled log-moneyness

$$x = \frac{1}{\sqrt{T}} \log \frac{K}{F} \qquad \text{with} \qquad \begin{aligned} K &= \text{option strike} \\ F &= \text{underlying futures price (equal to forward), and} \\ T &= \text{year fraction to option expiration date.} \end{aligned}$$

The horizontal axis in effect is a volatility: if σ is the volatility of the log price, then in time T the price will move approximately to $F \pm \sigma K \sqrt{T}$. With this scaling for the log-moneyness, the curves for different maturities approximately superimpose.

We use daily option settlement prices from the CME end-of-day (EOD) data product. We calculate implied volatility using the Black model with zero interest rate and actual/actual day count. This agrees precisely with implied volatility values provided by CME, except that CME truncates its calculation for deep out-of-the-money contracts. We reject the very small number of data values that do not satisfy put-call parity, and we exclude options with less than 7 days to maturity.

The biggest difference between the upper two graphs of Figure 2 and the lower two is the asymmetry of the upper graphs, and the symmetry of the lower graphs.

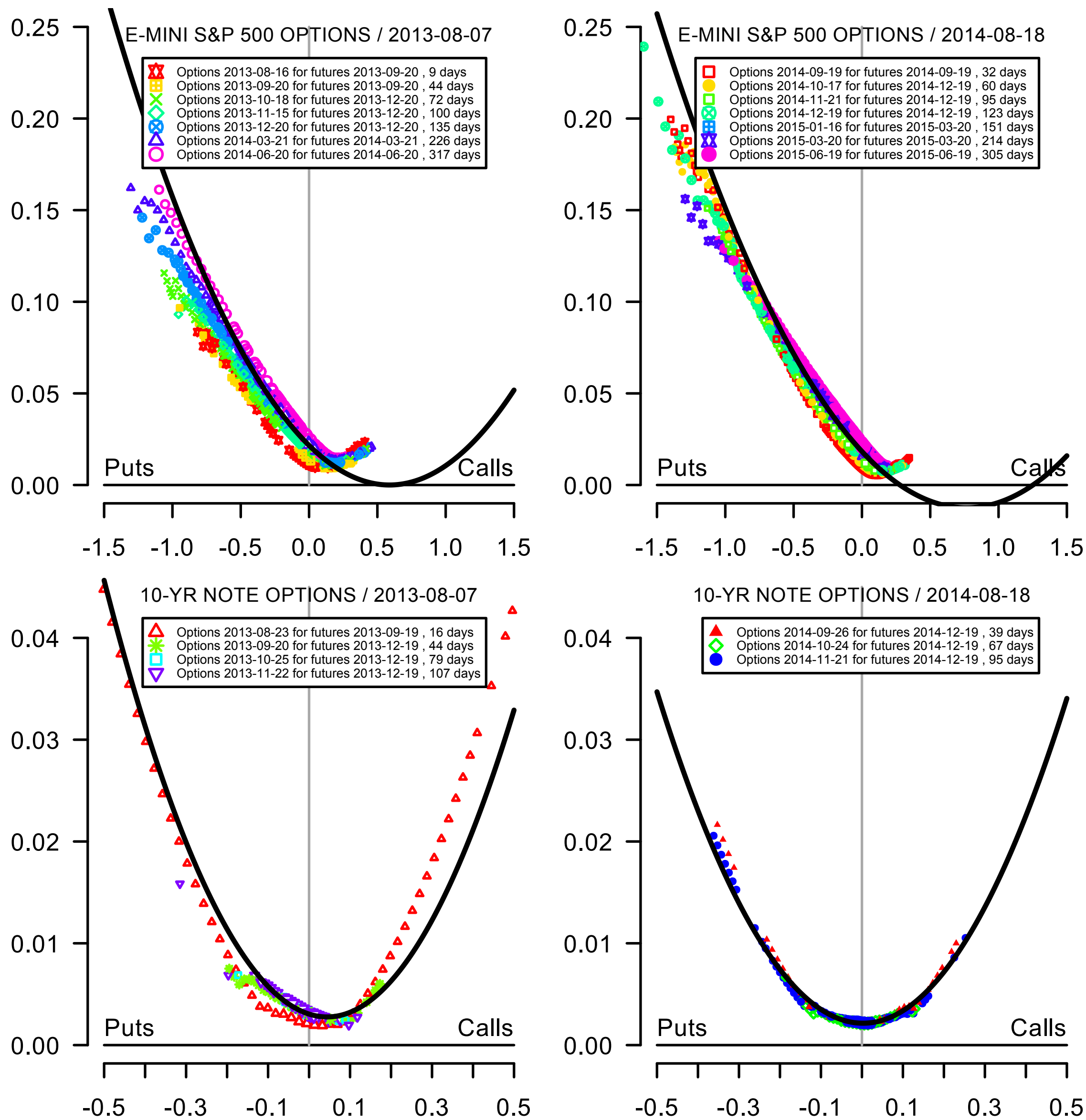

Figure 2: Implied variance $v = \sigma_{\mathrm{BS}}^2$ (square of Black-Scholes implied volatility, in annual percent) for options on the S&P 500 equity index and 10-year Treasury futures. Left column is a typical date in 2013, right column in 2014. Horizontal axis is scaled log-moneyness $x = \log(K/F)/\sqrt{T}$. The labels "Calls" and "Puts" denote which contract is cheap, hence most actively traded, in that range of strikes. Lines represent a weighted quadratic fit to the full set of strikes and expirations, for the purpose of estimating volatility and its derivatives around the money $x = 0$. SP500 options have consistent negative skew throughout the period. Treasury options had moderate negative skew in 2012 and 2013, and essentially no skew in 2014.

The asymmetry in implied volatility for SP500 options is well-known, and is generally taken to represent a fear that equity market prices will fall. There is high demand for out-of-the money puts, whose strike is below spot, and hence the implied volatility is driven up for those products; also a much wider range of strikes is traded on that side. There is less demand for out-of-the money calls, and their implied volatility is typically lower than the implied volatility at the money. The implied volatility does rise again for out-of-the-money calls, but the available range of strikes is very short.

For Treasury options, a wide range of strikes is traded on both sides of the spot, especially at short maturities. The graph is nearly symmetric, especially for the picture on the right showing a representative date in 2014. On the left, a slight skew is evident, but it is much less significant than in the upper row. The most visible shape feature is the increase in implied volatility on both sides, represents fat tails in the return distribution rather than a particular economic belief.

Quadratic fit The black lines in Figure 2 represent a quadratic fit of the form

$$v = a + bx + \frac{1}{2}cx^2.$$

Since our goal is to extract volatility and its derivatives at the money $x = 0$, we weight each data point by the square of the Black-Scholes vega, multiplied by a "concentration factor" $\exp(-x^2/2x_0^2)$ where x_0 is an estimate of the volatility: 0.05 for 10-Year Treasury and 0.15 for SP500. The fit also works well with no weighting, but the coefficients show conspicuous jumps as an expiration approaches and certain maturities drop out of our data set. Because we use our fit only around $x = 0$, it is not significant that the fitted value of σ^2 become negative for large x in the upper panels.

Thus $a = v(0)$ is the at-the-money variance, and $\sigma_{\text{BS}}(0) = \sqrt{a}$ is the at-the-money implied volatility. These values are graphed in Figure 3. The values computed by this method are consistent with the values of the CBOE VIX and VXTYN contracts, which estimate an implied volatility by a completely different method. The implied volatility of the SP500 options are about three times as large as the implied volatility of the 10-Year Treasury options: 15% annual for SP500 *vs.* 5% annual for Treasury.

The coefficient b is a dimensional value for the at-the-money variance skew $d\sigma_{\text{BS}}^2/dx\big|_{x=0}$. From Figure 2 we expect this parameter to be much smaller for the 10-Year Treasury options than for the SP500 options. But to make a precise comparison we must determine a suitable scaling.

Nondimensionalization As noted above, the volatility value itself corresponds to a value of the scaled log-moneyness x. This suggests that we define two nondimensional variables

$$\phi = \frac{v}{v(0)} = \frac{v}{a}, \qquad \xi = \frac{x}{X}, \qquad \text{with} \qquad X = \sigma_{\text{BS}}(0) = \sqrt{a}.$$

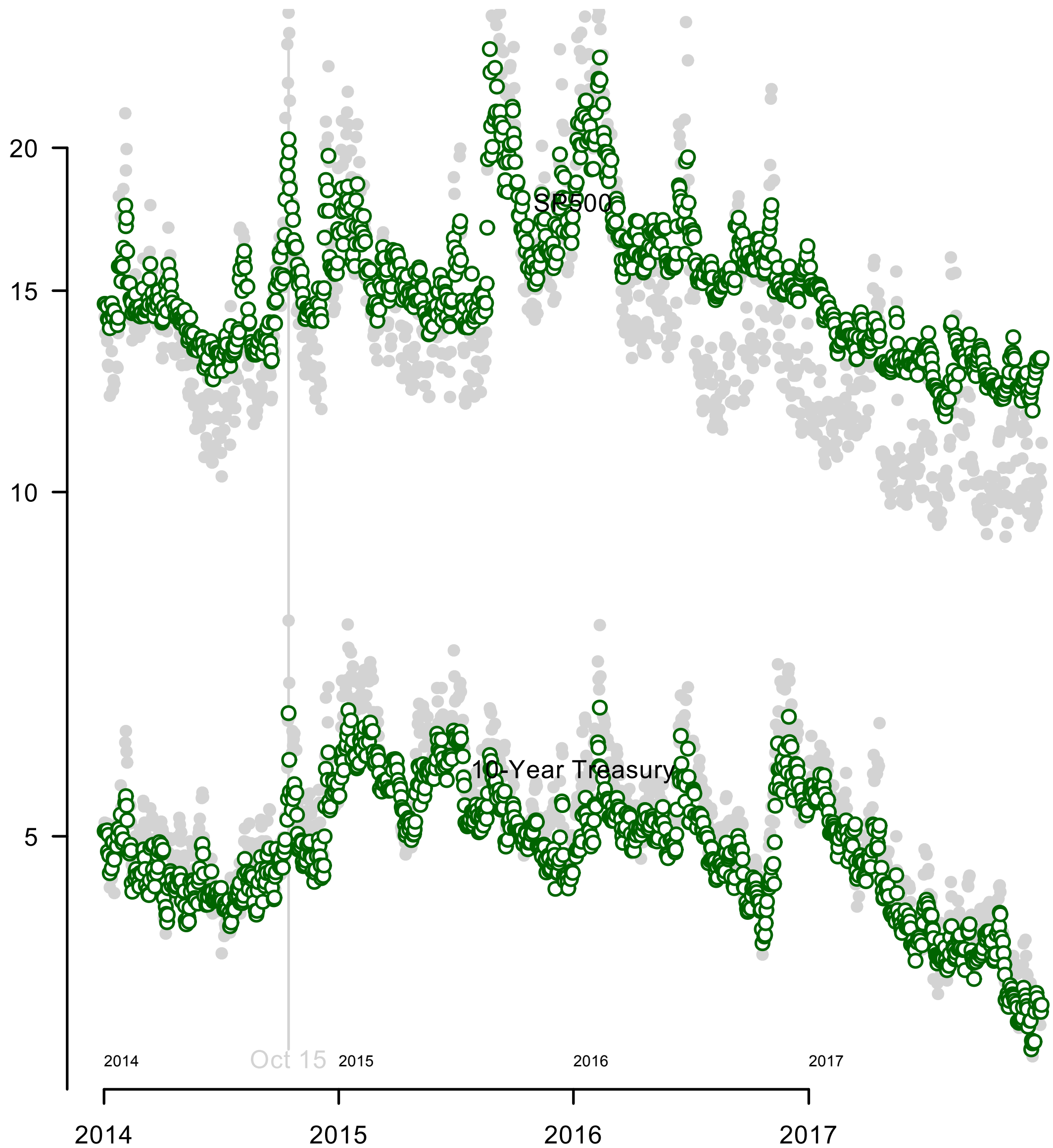

Figure 3: At-the-money implied volatilities $\sigma_{\mathrm{BS}}(0)$, in percent annual on a logarithmic scale, for options on SP500 futures and options on 10-Year US Treasury futures. Circles are as determined by the quadratic fit to the surface as illustrated in Figure 2. Light gray dots are values of the CBOE VIX and VXTYN contracts, for comparison. At-the-money implied volatility for the SP500 is around 15% annual. For the 10-Year Treasury it is around 5% annual but rising into early 2015; the rise in summer–fall 2013 reflects the "taper tantrum." Both implied volatilities rise before Oct. 15, 2014, as we discuss in the text.

In terms of these nondimensional variables, the quadratic fit has the form

$$\phi = 1 + B\xi + \frac{1}{2}c\,\xi^2 \qquad \text{with} \qquad B = \frac{b}{\sqrt{a}}.$$

The coefficient on the quadratic term is unchanged: it is inherently nondimensional since the x variable has units of volatility and thus x^2 has the same units as v.

The value B is the answer to this question: Across a time interval of one year, the relevant range of strike is about $x = \pm\sigma_{\rm BS}(0) = \pm\sqrt{a}$. This range corresponds to a change in implied variance of about $\pm b\sqrt{a}$. What is the magnitude of this change, as a fraction of the implied variance a at $x = 0$? The answer is $B = b\sqrt{a}/a = b/\sqrt{a}$. It is this nondimensional number that we should compare between different products.

Figure 4 shows this nondimensional skew coefficient for SP500 and for 10-Year Treasury options. The skew for the former is small in nondimensional terms compared with the latter. For example, in 2013 the nondimensional skew of Treasury options was around -0.3, while for SP500 options it was around -0.5.

At-the-money skew for Treasury options increased steadily toward zero through 2014, even becoming slightly positive in late 2014 and early 2015. In our opinion, this represents the market's diminishing fear of an interest rate rise, which would cause bond prices to fall. This fear has reappeared fairly suddenly in the second quarter of 2015.

This coefficient B is also the at-the-money volatility skew. Indeed,

$$b = \frac{dv}{dx} = \frac{d\sigma^2}{dx} = 2\sigma\frac{d\sigma}{dx} \qquad \text{so} \qquad \frac{d\sigma}{dx} = \frac{b}{2\sigma} = \frac{b}{2\sqrt{a}}$$

where $\sigma = \sigma_{\rm BS}$ and all functions are evaluated at $x = 0$.

The quadratic coefficient is the second derivative

$$c = \frac{d^2v}{dx^2} = \frac{d^2\sigma^2}{dx^2} = \frac{d}{dx}\left(2\sigma\frac{d\sigma}{dx}\right) = 2\left(\left(\frac{d\sigma}{dx}\right)^2 + \sigma\frac{d^2\sigma}{dx^2}\right).$$

With $d\sigma/dx = b/2\sqrt{a}$ this gives

$$\frac{d^2\sigma}{dx^2} = \frac{1}{2\sqrt{a}}\left(c - \frac{b^2}{2a}\right).$$

A nondimensionalized version of this (recall that σ has the same units as x) is

$$C = \sigma\frac{d^2\sigma}{dx^2} = \frac{1}{2}\left(c - \frac{b^2}{2a}\right).$$

Figure 5 shows this nondimensional volatility curvature C for 10-Year US Treasury options only. The values for the SP500 options are substantially smaller than those for the 10-Year Treasury option, but we do not rely on this fact (in fact, the variance skew c has values of similar magnitude for both contracts). Rather, we argue that the leading-order part of the local behavior is captured by the first coefficient that is significantly different from zero. For equity index options this is the skew. For Treasury options, at least in 2014, it is the curvature.

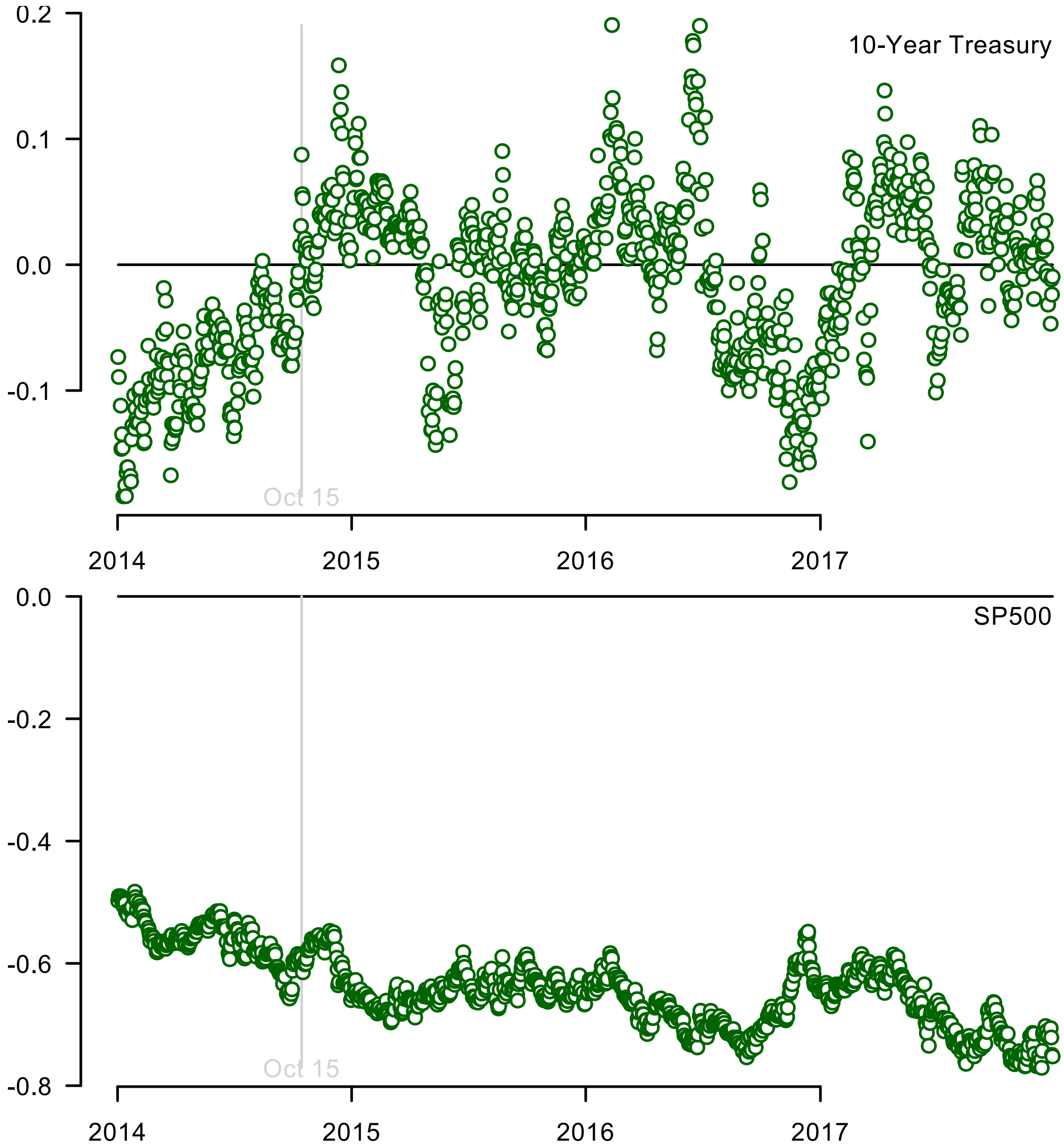

Figure 4: Nondimensional at-the-money implied skews *B* as described in the text. As expected from Figure 2, the values for the SP500 options are consistently large and negative. For the 10-Year Treasury option, the values are smaller in magnitude, and have reversed from negative to near zero in 2014. In early 2015 they are returning to negative, signaling market fear of an interest rate rise.

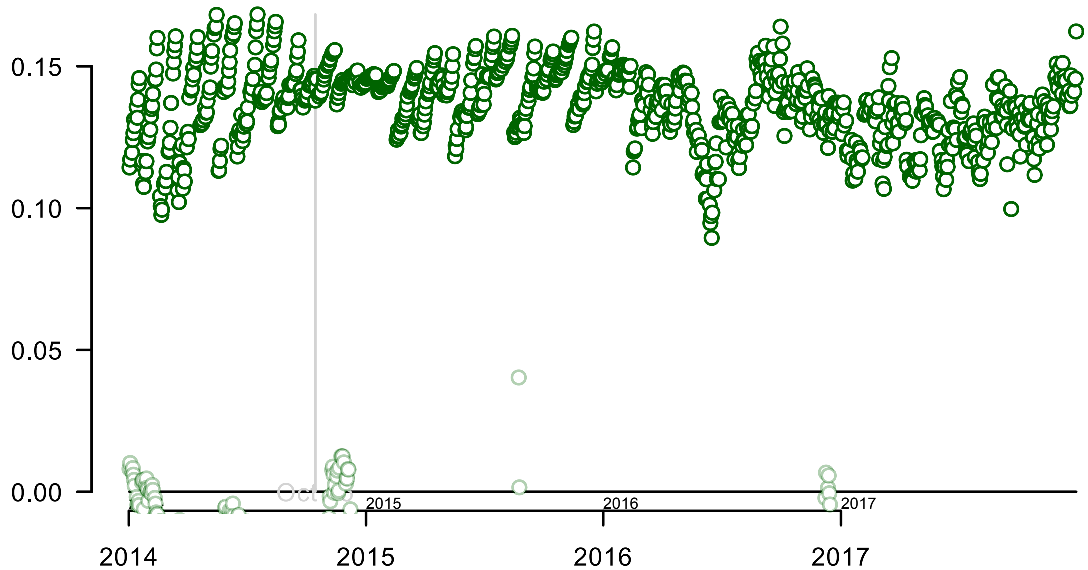

Figure 5: Nondimensional at-the-money volatility curvature for 10-Year Treasury options, which is consistently substantial and positive. The faint values are for the SP500 equity index futures, for which the curvature is less significant than the skew.

2 October 15

On the morning of October 15, 2014, between 9:35 and 9:45 New York time, yields on US Treasury securities underwent their largest single-day drop since 2009, and quickly recovered. For example, the yield on the 10-year note fell from 2.05% to 1.873% before recovering. Articles in *The Wall Street Journal*[1], *Financial Times*[2], and *Bloomberg*[3] have suggested that these sharp price moves may have been in part due to the increasing prevalence of electronic trading, which is destabilizing the market for US Treasury cash products and futures.

The most detailed study of this event appeared in *Risk* magazine.[4] Based on conversations with 24 banks, hedge funds, and others, *Risk* reported that in the first weeks of October, the sell-side as a whole was accumulating a substantial short gamma position. Hedging of these positions created market instability and amplification of any price moves.[5] This suggests that the sharp price moves were a consequence of fundamental asset dynamics rather than electronic trading and intrinsic market structure.

[1] Tom Lauricella and Katy Burne, "Bond Swings Draw Scrutiny," *Wall Street Journal*, Nov. 9, 2014.

[2] Tracy Alloway and Michael MacKenzie, "Bonds: Anatomy of a market meltdown," *Financial Times*, Nov. 17, 2014.

[3] Susanne Walker and Lisa Abramowicz, "Flash boys raise volatility in wild new Treasury market," *Bloomberg*, Nov. 18, 2014.

[4] Kris Devasabai, "No flash crash: Paulson, Pimco and the US Treasury meltdown," *Risk*, Dec. 8, 2014.

[5] See, *e.g.*, T. M. Li and R. Almgren, "Option hedging with smooth market impact," preprint 2014.

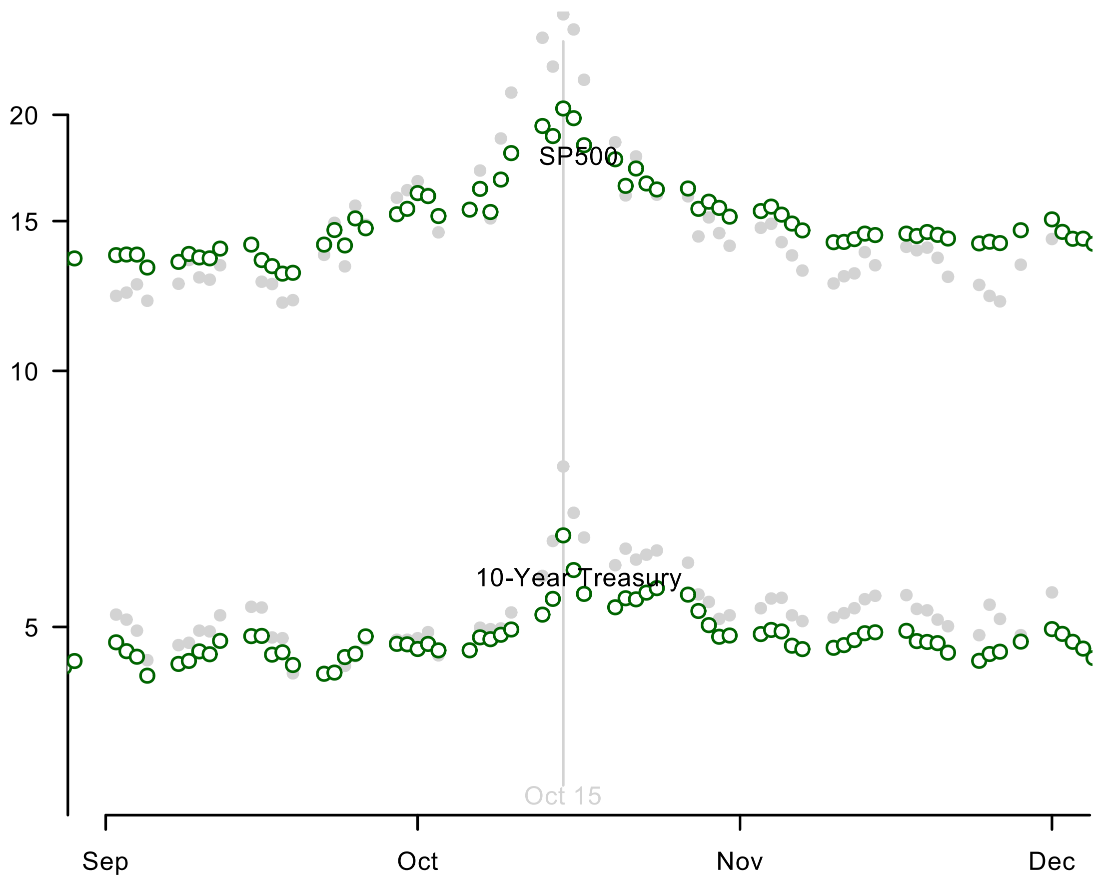

Figure 6: At-the-money implied volatilities $\sigma_{BS}(0)$, as in Figure 3, for a 3-month window centered on October 15, 2014 (gray dots are VIX and VXTYN). Equity implied volatility rises for two weeks before Oct. 15, with a lesser rise in Treasury implied volatility, suggesting that the markets in general were coming under stress.

The data in Figure 6 support this interpretation. We see a smooth steady rise in equity index implied volatility in the weeks leading up to Oct. 15, and a smooth steady decline in the weeks following. This suggests that there was pressure on the equity market, but that nothing special happened on Oct. 15 itself.

Treasury implied volatility rose very slightly before Oct. 15, but was high on that day in particular. This suggests that the Treasury price move was a response to pressure in the equity market rather than an intrinsic behavior driven by electronic trading of Treasuries.