

Adjusted Interest Rate (AIR) Total Return Futures Explained



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Introduction

The Adjusted Interest Rate Total Return futures (AIR TRF) contract is designed to offer similar economics to an equity index swap. Before examining the characteristics of the AIR TRF, it is helpful to look at the mechanics of an equity index swap.

If one is already familiar with swap mechanics, please move directly to section 2 on AIR TRF mechanics. Sections 3, 4, and 5 detail valuation, daily, and final settlement, respectively. Section 7 presents the contract specifications.

1. Equity index swap mechanics

In an equity index swap, one counterparty agrees to pay the other index performance in exchange for an agreed financing payment. The most common swap structures are designed with the periodic exchange of payments with corresponding resets of interest rate and index notional values. The financing payment for the period is based on the notional value of the equity index exposure and uses the agreed interest rate reference rate that aligns to the period length, such as 3-month Libor for quarterly resets, or the overnight Effective Fed Funds Rate (EFFR) for daily resets +/- an agreed spread.

Please note that the agreed-upon spread does not change during the tenor of the swap; the spread is a fixed differential applied to the periodic-resetting reference rate where the net effective interest rate is used each period to calculate the financing payment.

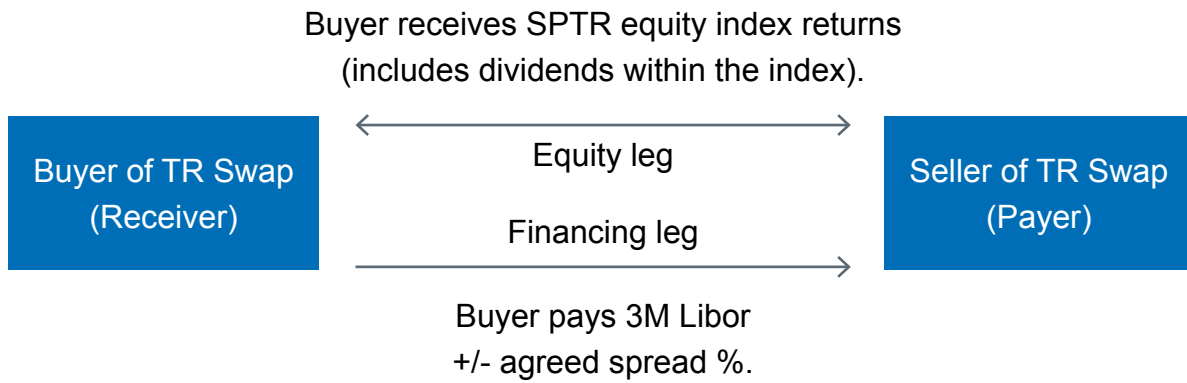
The counterparty that is long equity performance (Receiver) also receives any dividend income from the counterparty who is short equity performance (Payer). If the underlying index is a total return index, the dividends are reinvested in the underlying index as part of the index methodology, and an explicit adjustment for dividends need not be made. In the case of a swap based on a price return equity index, the dividend adjustment will be required either through scheduled dividend payments or an adjustment to the agreed-upon financing spread.

The Receiver of equity performance makes financing payments to the Payer of equity performance to compensate the Payer for the financing costs associated with the funding the underlying equity position (or hedge).

At the agreed periodic intervals (swap reset dates), cash flows are exchanged between the counterparties based on the economics of the various swap legs, i.e., the equity leg, the financing leg, and dividend leg (if applicable). On the swap reset dates, the index notional and reference rate are adjusted each time to account for the exchange of cash flows. The newly observed level for both the equity notional and the reference rate will apply to the subsequent period until the next reset.

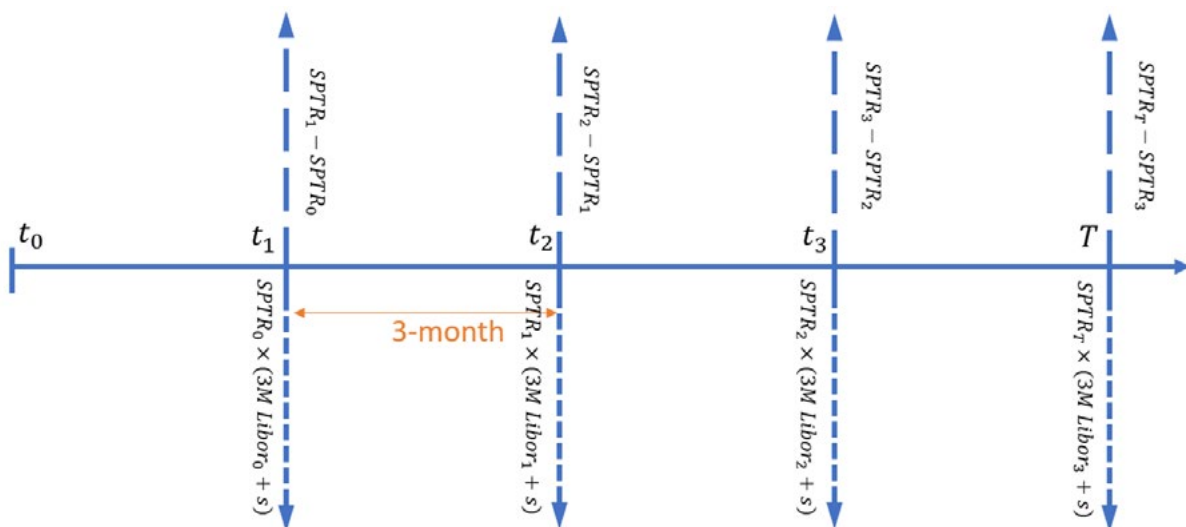
Thus, the equity index swap mechanics can be summarized in Figure 1 below based on the S&P 500 Total Return index (SPTR) and using 3-month Libor as the reference rate for the financing leg.

Figure 1: Equity swap mechanics



Based on a one-year swap with quarterly resets, the cash flows determined at resets will be as follows: Where $SPTR_t$ is the underlying index price at time t . t_0 is the trade inception, T is the maturity and the reset date intervals are at t_i , $3M\ Libor_i$ is the floating reference rate for the period, s is the financing spread (which is fixed) quoted in basis points, and r is the floating reference rate. Since r and s are yearly rates, they need to be multiplied by the year fraction τ .¹ In Figure 2 below, the counterparty long the equity performance will receive the cash flows determined by the top half of the diagram, whilst paying the cash flows in the lower half of the diagram.

Figure 2: Cash flows of an equity swap



¹ This is ACTUAL/360. For simplification purposes quarterly could be assumed to be $\tau = \frac{90}{360}$

2. Adjusted Interest Rate Total Return futures mechanics

The AIR TRF has a maturity at a known date and its valuation has three components: an equity index component, a benchmark financing component that accrues daily, and a financing spread adjustment component.

$$\text{AIR TRF} = (\text{Equity Index} - \text{Accrued Financing}) + \text{Financing Spread Adjustment}$$

The equity index component of an AIR TRF is similar to a swap in that it is an agreement between two counterparties to pass equity performance from one counterparty to the other. The equity index price of the AIR TRF is always the official index daily close, as the trade is transacted via BTIC.²

The next component of the AIR TRF is the sum of the daily accrued financing (AF). Compared to an equity swap, where the financing amount accrued is exchanged at each reset date as a cash flow; in the AIR TRF the financing amount is accrued daily based upon the benchmark reference rate (e.g. the Effective Fed Funds Rate, or EFFR) and the sum is incorporated into the daily settlement of the product. For each contract the initial value for AF will be determined by the exchange and all the daily financing amounts will be added to this initial value (see formulae in section 3).³

This accrued amount is netted with the equity index performance as shown in the formula above. Thus, the buyer of the AIR TRF will have the exposure of the equity index minus the sum of the accrued daily financing to date. Therefore, the buyer's exposure will incur a negative carry if this accrued financing amount is positive. Conversely, the seller will earn this positive carry.

The last component of the AIR TRF valuation is the financing spread adjustment. When trading the AIR TRF, the counterparties will agree to lock in a spread +/- to the reference rate (TRF Spread) for the remaining maturity of the product. The driver of determining this spread rate will be the rebate value the market ascribes to the value of the underlying stocks in the index.

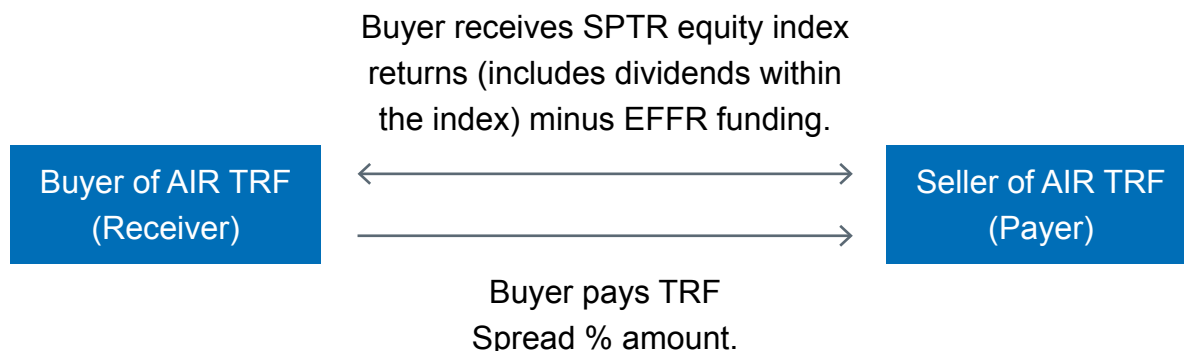
Thus, the spread that is agreed to is equivalent to the spread that would be charged above or below the reference rate in an equity index swap. As is observed with swap spread pricing, the higher the rebate value, the lower the spread of the AIR TRF, and vice versa. The component that will be most impactful on the price discovery of AIR TRFs will be this financing spread adjustment component.

² The only exception is trading the future via an EFRP where the future price level is specified directly.

³ The AF will be the same across all maturities regardless of listing date. The differential of the AF between trade entry and exit will be important to determining how the valuation evolves, not the initial value.

The daily economics of the AIR TRF based on a total return index are summarized in Figure 3 below.

Figure 3: AIR TRF mechanics



3. AIR Total Return futures valuation

Market participants will trade the TRF spread price (s_t) directly. Once the TRF spread is consummated, it will be converted into an AIR TRF price by the Exchange. The resulting cleared price of the future is computational and occurs on a trade by trade basis as outlined below. The **quoting notation** will be the TRF spread expressed in basis points (rather than index points). The TRF spread will be available to trade in 0.5 basis point increments.

The price of AIR TRF is defined as:

$$\begin{aligned}
 \text{Future Price}_t &= (\text{Equity Index} - \text{Accrued Financing}) + \text{Financing Spread Adjustment} \\
 &= (SPTR_t - AF_t) + FSA_t \\
 &= (SPTR_t - AF_t) + SPTR_t \times \tau_t \times s_t
 \end{aligned}$$

Where:

- t : valuation time;
- $SPTR_t$: S&P 500 Total Return index close price. It is the Gross Total Return index on S&P 500 also known by the index ticker "SPTR";
- AF_t : Accrued daily overnight financing at time t defined by $AF_t = AF_{t-1} + DF_t$;
- DF_t : Daily financing at time t as defined by $DF_t = SPTR_{t-1} \times EFFR_{t-1} \times \tau_t^{FD}$;
- FSA_t : Financing spread adjustment as defined by $FSA_t = SPTR_t \times \tau_t \times s_t$;
- s_t : TRF spread price;
- T : Expiration date of the future;
- τ_t^{FD} : annualized financing days as defined by $\tau_t^{FD} = [(t)+2 \text{ settlement days}] - [(t-1) + 2 \text{ settlement days}] / 360$.
- τ_t : time to expiry defined by $\tau_t = [(T+ 2 \text{ settlement days}) - (t + 2 \text{ settlement days})] / 360$.

The overnight financing rate is determined as the Effective Fed Funds Rate "EFFR" applicable to each day. Accrued financing (AF_t) is equal to the sum of the daily overnight financing (DF_t) since the product's launch on Sep 21, 2020 until the valuation time (t): $AF_t = \sum_{i=t_0}^t SPTR_{i-1} \times EFFR_{i-1} \times \tau_i^{FD}$. The DTCC business days calendar is used as the "settlement days" calendar to calculate the time to expiry (τ_t) and the annualized financing days (τ_t^{FD}).

A fictitious **numerical example** of the price of the valuation and profit and loss calculation can be found below using the following input values:

Table 1: Example daily closing values for SPTR, EFFR, and TRF spread

	SPTR	EFFR (%)	TRF spread daily settlement (bps)
9/16/2020	6,600.00	1.54	N/A
9/17/2020	6,610.19	1.54	20
9/18/2020	6,650.93	1.54	19.5

It is assumed the AIR TRF is listed on September 17 and has a maturity of December 18, 2020. The TRF spread traded during the day by an investor is assumed to be **18.5bps**.

The September 17 and 18, 2020 valuations are calculated to determine the profit and loss value between the initial trade and the two valuation dates (further detailed calculations are available in Appendix 2 & 4).

On September 17, the daily funding (DF) is equal to 0.847 because the previous SPTR close and EFFR were 6,660 and 1.54 percent, respectively, and the funding days are equal to 3: $6,660 \times \frac{3}{360} \times \frac{1.54}{100} = 0.8470$.

Furthermore, the accrued financing (AF) is also 0.847 because the contract was just listed on that day and the initial value of AF determined by the exchange was zero. The traded financing spread adjustment (FSA) is 3.1252 because SPTR closed at 6,610.19, the TRF Spread traded during the day was 18.5bps and the time-to-maturity is 92 days: $6,610.19 \times \frac{18.5}{10,000} \times \frac{92}{360} = 3.1252$. Therefore, the AIR TRF valuation at the time of trading is = $6,610.19 - 0.8470 + 3.1252 = \mathbf{6,612.47}$.

For daily settlement on September 17, the TRF spread settled at 20 bps, making the FSA value for the daily settlement of the AIR TRF 3.3785 ($6,610.19 \times \frac{20}{10,000} \times \frac{92}{360}$) and resulting in a futures settlement value of **6,612.72**. The investor's profit on the 17th is **0.25** index points per contract, which at the \$25 contract multiplier would make the total profit per contract \$6.25.

The same calculations are computed for September 18, 2020. The previous SPTR close and EFFR were 6,610.19 and 1.54 percent, respectively, and one day of financing occurred. Thus, the daily financing is 0.2828: $6,610.19 \times \frac{1}{360} \times \frac{1.54}{100}$. The accrued financing is 1.1298, which is the sum of the two days daily financing.

The FSA is 3.2784 because SPTR closed at 6,650.93, the daily settlement of the TRF-settled spread decreased to 19.5 and the time-to-maturity is now 91 days: $6,650.93 \times \frac{19.5}{10,000} \times \frac{91}{360} = 3.2784$.

Thus, the AIR TRF daily settlement on the 18th is = $6,650.93 - 1.1298 + 3.2784 = \mathbf{6,653.08}$ and the additional profit for September 18 is **40.36** index points per contract, worth \$1,009.00 (\$25 multiplier), which brings the total two-day profit to \$1,015.25 per contract.

It is important to note that only final valuation values and profit and loss are rounded during the calculation. All intermediary results' precision is used in the next steps calculation.

4. Daily settlement of AIR Total Return futures

The daily settlement price of AIR Total Return futures will be based on the computation of the theoretical value of the contracts using:

- Daily closing index value of the total return index;
- Daily EFFR rate;
- Daily TRF Spread settlement based on quoted market, or VWAP of traded prices during the session, or previous spread settlement if no current day activity (methodology subject to further refinement).

The daily settlement price of the contract shall be determined based on the following formula:

$$\begin{aligned} \text{Futures Price}_t^{\text{daily settle}} &= (\text{Equity Index} - \text{Accrued Financing}) + \text{Financing Spread Adjustment} \\ &= (SPTR_t - AF_t) + FSA_t^{\text{settle}} \\ &= (SPTR_t - AF_t) + SPTR_t \times \tau_t \times s_t^{\text{settle}} \end{aligned}$$

where the Spread Settle (s_t^{settle}) for the day shall be determined based on market activities (quotes, traded price) or prior day settle if no market activities, and AF_t is the sum of accrued daily overnight financing until settlement date.

Appendix 1 shows the profit and loss definition and how the daily variation margin (VM) would work. It demonstrates the fact that it replicates the daily performance of the STPR less the daily overnight financing as well as any profit or loss in the financing spread adjustment. Appendix 2 details numerical examples of both valuation and VM.

5. Final settlement of AIR Total Return futures

The final settlement of the AIR TRF is the SPTR SOQ price minus the sum of accrued daily overnight financing until expiry. The term FSA_T at final expiration is null because at expiration of the future τ_T is zero.

$$\text{Future Price}_{t=T}^{\text{final settlement}} = SPTR_T^{\text{SOQ}} - AF_T$$

The expiry level of SPTR on final settlement will be determined by the Special Opening Quotation (SOQ) of the index together with the sum of the accrued financing up to and including the final settlement date.

6. Trading mechanism

AIR Total Return futures are tradable via spread-quoted BTIC only (Globex and ex-pit). Minimum Block size 500.

CONTRACT SPECIFICATIONS

Contract unit	\$25 x AIR S&P 500 Total Return Index Price
Underlying index	S&P 500 Total Return Index (SPTR)
Reference rate	Effective Fed Funds Rate (EFFR)
Trading quotation	TRF spread in basis points expressed as an annualized number.
Trading hours	CME Globex: BTIC: Sunday – Friday 6:00 p.m. – 4:00 p.m. Eastern Time (ET) CME ClearPort: BTIC: Sunday – Friday 6:00 p.m. – 4:00 p.m. ET
Minimum price fluctuation	0.5 Basis Points in terms of TRF Spread The resultant cleared AIR TRF future price will be rounded to 2 decimals.
Product code	CME Globex: ASR CME ClearPort: ASR Clearing: ASR BTIC: AST ASR is not tradable (except for EFRPs). The price basis for all CME Globex or block transactions shall be BTIC only.
Listed contracts	Quarterly contracts listed for 13 quarters and 4 additional December contract months
Settlement method	Financially settled
Termination of trading	Trading terminates on the 3rd Friday of the contract month. BTIC: Trading terminates on the business day prior to 3rd Friday of the contract month.
Settlement procedures	Daily settlement price of contract shall be determined based on the following formula: $= (SPTR_t - AF_t) + SPTR_t \times \tau_t \times S_t^{settle}$ where the <i>Spread Settle</i> (S_t^{settle}) for the day shall be determined based on market activities (quotes, traded price) or prior day settle if no market activities, and AF_t is the sum of accrued daily overnight financing until settlement. Final settlement price shall be determined based on the following formula: $SPTR_T^{SOQ} - AF_T$
Block minimum	500

8. Appendix 1 - Profit and loss definition

One can define the profit and loss, or cash variation margin, at time t as the summation of three terms:

$$\begin{aligned} P\&L_t^{AIR TRF} &= P\&L Equity + P\&L financing + P\&L Financing Spread Adjustment \\ &= (SPTR_t - SPTR_{t-1}) - (SPTR_{t-1} \times EFFR_{t-1} \times \tau_t^{FD}) \\ &\quad + (SPTR_t \times \tau_t \times s_t - SPTR_{t-1} \times \tau_{t-1} \times s_{t-1}) \\ &= \Delta SPTR - (SPTR_{t-1} \times EFFR_{t-1} \times \tau_t^{FD}) \\ &\quad + SPTR_{t-1} \times s_{t-1} \times \Delta \tau + SPTR_{t-1} \times \tau_t \times \Delta s + s_{t-1} \times \tau_t \times \Delta SPTR \\ &\quad + \tau_t \times \Delta SPTR \times \Delta s \end{aligned}$$

It is important to note that the *P&L financing* term is equal to the daily financing at time t.

Additionally, the last term, *P&L Financing Spread Adjustment* can be extended further into 4 terms⁴:

Daily financing spread paid, spread risk, (additional) equity risk and cross equity-spread risk.

One can analyze each term to see its impact on the risk management of the futures contract.

The first term, $SPTR_{t-1} \times s_{t-1} \times \Delta \tau$, represents the daily financing spread paid and is known at t-1. It is typically a relatively small⁵ daily loss⁶ for the buyer of the contract. The second term, $SPTR_{t-1} \times \tau_t \times \Delta s$, characterizes the spread risk with notional $SPTR_{t-1} \times \tau_t$ which is known at time t-1. The third term, $s_{t-1} \times \tau_t \times \Delta SPTR$, has a risk exclusively realized by the change in the equity index price and is also known at time t. The last term $\tau_t \times \Delta SPTR \times \Delta s$ is the cross equity-spread risk term and should be negligible.

In conclusion, the *P&L Financing Spread Adjustment* is mostly a function of the change in spread value if one manages the equity index risk.

9. Appendix 2 – Numerical example

This section goes through a hypothetical example for calculating the valuation and profit and loss of the AIR TRF.

It looks at a valuation from September 17-22, 2020. The futures contract expires in December 2020 and has a maturity date of December 18. It is assumed to have been listed on September 17, 2020. Table 2 shows the input data.

⁴ See appendix 3 for derivation

⁵ s_t is in basis point which should make the term relatively small compared to the others. Also, in the event this spread is negative than the *daily financing spread paid* would become a gain for the buyer of the contract.

⁶ $\Delta \tau$ is negative.

Table 2: Input data – Date, SPTR close price, EFR rate, TRF spread.

	SPTR	EFR (%)	TRF spread daily settlement (bps)
9/16/2020	6,600.00	1.54	N/A
9/17/2020	6,610.19	1.54	20
9/18/2020	6,650.93	1.54	19.5
9/21/2020	6,650.93	1.54	25
9/22/2020	6,650.93	1.54	25

Table 3 presents intermediary results for all variables in the TRF formula and valuation at different time.

Table 3: Intermediary results and valuation at different time.

	τ	τ^{FD}	DF	AF	FSA	$V^{AIR TRF}$
9/17/2020	92/360	3/360	0.8470	0.8470	3.3785	6,612.72
9/18/2020	91/360	1/360	0.2828	1.1298	3.2784	6,653.08
9/21/2020	90/360	1/360	0.2845	1.4143	4.1568	6,653.67
9/22/2020	89/360	1/360	0.2845	1.6988	4.1106	6,653.34

Table 4 presents AIR TRF profit and loss and its breakdown into three components.

Profit and loss on the 18th shows the combined impact of the daily spread financing paid, the financing spread adjustment changing and the SPTR value changing. On the 21st, the price of SPTR is kept unchanged between the 18th and the 21st to show how other factors affect the profit and loss. Similarly, on the 22nd, both SPTR and the financing spread adjustment prices remain unchanged from the 21st to the 22nd to show the costs of keeping the contract if prices are stale.

Table 4: Profit and loss and its breakdown into three components as described in Appendix 1.

	<i>P&L Equity</i>	<i>P&L financing</i>	<i>P&L Financing Spread Adjustment</i>	<i>P&L^{AIR TRF}</i>
9/18/2020	40.74	- 0.2828	- 0.1002	40.36
9/21/2020	0	- 0.2545	0.8785	0.59
9/22/2020	0	- 0.2845	- 0.046	-0.33

Finally, table 5 goes in the details of the *P&L Financing Spread Adjustment* breakdown.

Table 5: *P&L Financing Spread Adjustment* broken down into 4 terms, as described in Appendix 1.

	<i>Daily Financing Spread Paid</i>	<i>Spread Risk</i>	<i>Equity Risk</i>	<i>Cross Risk</i>
9/18/2020	- 0.0367	-0.0835	0.0206	-0.0005
9/21/2020	-0.0360	0.9145	0	0
9/22/2020	-0.0462	0	0	0

10. Appendix 3 – Profit and loss derivation:

$$\begin{aligned}
 \text{Financing Spread Adjustment P\&L} &= SPTR_t \times \tau_t \times s_t - SPTR_{t-1} \times \tau_{t-1} \times s_{t-1} \\
 &= (SPTR_{t-1} + \Delta SPTR) \times (\tau_{t-1} + \Delta\tau) \times s_t - SPTR_{t-1} \times \tau_{t-1} \times s_{t-1} \\
 &= SPTR_{t-1} \times \tau_{t-1} \times s_t + SPTR_{t-1} \times \Delta\tau \times s_t + \Delta SPTR \times \tau_{t-1} \times s_t + \Delta SPTR \times \Delta\tau \times s_t \\
 &\quad - SPTR_{t-1} \times \tau_{t-1} \times s_{t-1} \\
 &= SPTR_{t-1} \times \tau_{t-1} \times \Delta s + SPTR_{t-1} \times \Delta\tau \times (s_{t-1} + \Delta s) + \Delta SPTR \times \tau_{t-1} \times (s_{t-1} + \Delta s) \\
 &\quad + \Delta SPTR \times \Delta\tau \times (s_{t-1} + \Delta s) \\
 &= SPTR_{t-1} \times s_{t-1} \times \Delta\tau + SPTR_{t-1} \times \Delta s \times (\tau_{t-1} + \Delta\tau) + \Delta SPTR \times s_{t-1} \times (\tau_{t-1} + \Delta\tau) + \\
 &\quad \Delta SPTR \times \Delta s \times (\tau_{t-1} + \Delta\tau) \\
 &= SPTR_{t-1} \times s_{t-1} \times \Delta\tau + SPTR_{t-1} \times \tau_t \times \Delta s + s_{t-1} \times \tau_t \times \Delta SPTR + \tau_t \times \Delta SPTR \times \Delta s \\
 &= \text{Spread Financing Paid Daily} + \text{Spread Risk} + \text{Equity Risk} + \text{Cross Risk}
 \end{aligned}$$

11. Appendix 4 – Detailed calculation from the numerical example:

This section examines the valuation on the 17th and 18th as well as the profit and loss calculation on the 18th.

Valuation on the 17th (time t)

$$\tau_t = (\{12/18/2020 + 2 \text{ settlement days}\} - \{09/17/2020 + 2 \text{ settlement days}\})/360 = (\{12/22/2020\} - \{09/21/2020\})/360 = 92/360$$

$$\tau_t^{FD} = (\{09/17/2020 + 2 \text{ settlement days}\} - \{09/16/2020 + 2 \text{ settlement days}\})/360 = (\{09/21/2020\} - \{09/18/2020\})/360 = 3/360$$

$$DF_t = SPTR_{t-1} \times \tau_t^{FD} \times \frac{EFFR_{t-1}}{100} = 6,660 \times \frac{3}{360} \times \frac{1.54}{100} = 0.8470$$

$$AF_t = AF_{t-1} + DF_t = 0 + 0.8470 = 0.8470$$

$$FSA_t = SPTR_t \times \frac{s_t}{10,000} \times \tau_t = 6,610.19 \times \frac{20}{10,000} \times \frac{92}{360} = 3.3785$$

$$V_t^{AIR TRF} = SPTR_t - AF_t + FSA_t = 6,612.72 \text{ (rounded to cent)}$$

Valuation on the 18th (time “t+1”)

$$\tau_{t+1} = (\{12/18/2020 + 2 \text{ settlement days}\} - \{09/18/2020 + 2 \text{ settlement days}\})/360 = (\{12/22/2020\} - \{09/22/2020\})/360 = 91/360$$

$$\tau_{t+1}^{FD} = (\{09/18/2020 + 2 \text{ settlement days}\} - \{09/17/2020 + 2 \text{ settlement days}\})/360 = (\{09/22/2020\} - \{09/21/2020\})/360 = 1/360$$

$$DF_{t+1} = SPTR_t \times \tau_{t+1}^{FD} \times \frac{EFFR_t}{100} = 6,610.19 \times \frac{1}{360} \times \frac{1.54}{100} = 0.2828$$

$$AF_{t+1} = AF_t + DF_{t+1} = 0.8470 + 0.2828 = 1.1298$$

$$FSA_{t+1} = SPTR_{t+1} \times \frac{s_{t+1}}{10,000} \times \tau_{t+1} = 6,650.93 \times \frac{19.5}{10,000} \times \frac{91}{360} = 3.2784$$

$$V_{t+1}^{AIR TRF} = SPTR_{t+1} - AF_{t+1} + FSA_{t+1} = 6,653.08 \text{ (rounded to cent)}$$

Profit and loss on the 18th (in index points per contract)

$$P\&L_{t+1}^{AIR\ TRF} = V_{t+1}^{AIR\ TRF} - V_t^{AIR\ TRF}$$

$$= (SPTR_{t+1} - SPTR_t) - (SPTR_t \times EFFR_t \times \tau_{t+1}^{FD}) + (SPTR_{t+1} \times \tau_{t+1} \times s_{t+1} - SPTR_t \times \tau_t \times s_t)$$

$$= 40.74 - 0.2828 + (-0.1002) = 40.36$$

Breaking down the traded basis P&L into its four components:

- $SPTR_t s_t \Delta\tau = -0.0367$
- $SPTR_t \tau_{t+1} \Delta s = -0.0835$
- $s_t \tau_{t+1} \Delta SPTR = 0.0206$
- $\tau_{t+1} \Delta SPTR \Delta s = -0.0005$

For more information about AIR Total Return futures, visit cmegroup.com/airtrf or contact a member of the Equity Products team at equities@cmegroup.com

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