



Sortino:
A 'Sharper' Ratio



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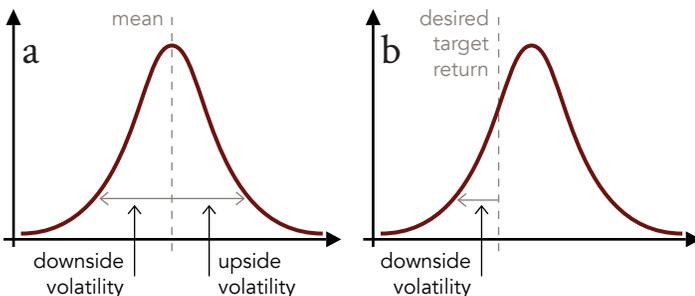
Many traders and investment managers have the desire to measure and compare CTA managers and / or trading systems. We believe risk-adjusted returns are one of the most important measures to consider since, given the inherent / free leverage of the futures markets, more return can always be earned by taking more risk. The most popular measure of risk-adjusted performance is the Sharpe ratio. While the Sharpe ratio is definitely the most widely used, it is not without its issues and limitations. We believe the Sortino ratio improves on the Sharpe ratio in a few areas. The purpose of this article, however, is not necessarily to extol the virtues of the Sortino ratio, but rather to review its definition and present how to properly calculate it since we have often seen its calculation done incorrectly.

Sharpe Ratio

The Sharpe ratio is a metric which aims to measure the desirability of a risky investment strategy or instrument by dividing the average period return in excess of the risk-free rate by the standard deviation of the return generating process. Devised in 1966 as a measure of performance for mutual funds, it undoubtedly has some value as a measure of strategy “quality”, but it also has a few limitations (see Sharpe 1994 for a recent restatement and review of its principles).

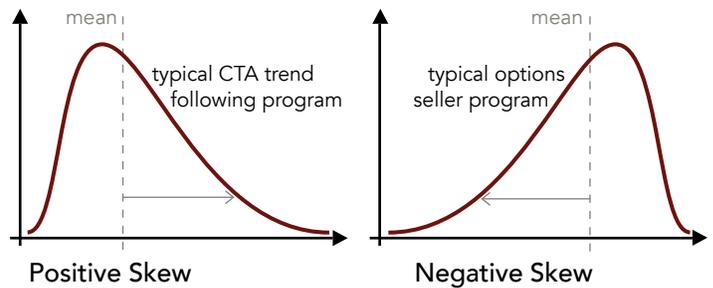
The Sharpe ratio does not distinguish between upside and downside volatility (GRAPH 1a). In fact, high outlier returns have the effect of increasing the value of the denominator (standard deviation) more than the value of the numerator, thereby lowering the value of the ratio. For a positively skewed return distribution such as that of a typ-

GRAPH 1



GRAPH 2

Same Sharpe, much different risk.



ical trend following CTA strategy, the Sharpe ratio can be *increased* by removing the largest positive returns. This is nonsensical since investors generally welcome large positive returns. To the extent that the distribution of returns is non-normal, the Sharpe ratio falls short. It is a particularly poor performance metric when comparing positively skewed strategies like trend following to negatively skewed strategies like option selling (GRAPH 2). In fact, for positively skewed return distributions, performance is actually achieved with *less risk* than the Sharpe ratio suggests. Conversely, standard deviation understates risk for negatively skewed return distributions, i.e. the strategy is actually *more risky* than the Sharpe ratio suggests.

Sortino Ratio

In many ways, the Sortino ratio is a better choice, especially when measuring and comparing the performance of managers whose programs exhibit skew in their return distributions. The Sortino ratio is a modification of the Sharpe ratio but uses downside deviation rather than standard deviation as the measure of risk—i.e. only those returns falling below a user-specified target (See “desired target return,” GRAPH 1b), or required rate of return are considered risky.

It is interesting to note that even Nobel laureate Harry Markowitz, when he developed Modern Portfolio Theory in 1959, recognized that since only downside deviation is relevant to investors, using downside deviation to measure risk would be more appropriate than using standard deviation. However, he used variance (the square of standard deviation) in his MPT work since optimizations using downside deviation were computationally impractical at the time.

In the early 1980s, Dr. Frank Sortino had undertaken research to come up with an improved measure for risk-adjusted returns. According to Sortino, it was Brian Rom's idea at Investment Technologies to call the new measure the Sortino ratio. The first reference to the ratio was in *Financial Executive Magazine* (August, 1980) and the first calculation was published in a series of articles in the *Journal of Risk Management* (September, 1981).

SORTINO RATIO

$$S = \frac{(R - T)}{TDD}$$

Dissecting the Ratio

The Sortino Ratio, S is defined as:

$$S = \frac{(R - T)}{TDD}$$

Where:

R = the average period return

T = the target or required rate of return for the investment strategy under consideration. In Sortino's early work, T was originally known as the minimum acceptable return, or MAR. In his more recent work, MAR is now referred to as the Desired Target Return™.

TDD = the target downside deviation. The target downside deviation is defined as the root-mean-square, or RMS, of the deviations of the realized return's underperformance from the target return where all returns above the target return are treated as underperformance of 0.

Mathematically:

Target Downside Deviation =

$$\sqrt{\frac{1}{N} \sum_{i=1}^N (\text{Min}(0, X_i - T))^2}$$

Where:

X_i = ith return

N = total number of returns

T = target return

The equation for TDD is very similar to the definition of standard deviation:

Standard Deviation =

$$\sqrt{\frac{1}{N} \sum_{i=1}^N (X_i - u)^2}$$

Where:

X_i = ith return

N = total number of returns

u = average of all X_i returns.

The differences are:

1. In the Target Downside Deviation calculation, the deviations of X_i from the user selectable *target return* are measured, whereas in the Standard Deviation calculation, the deviations of X_i from the *average* of all X_i is measured.

2. In the Target Downside Deviation calculation, all X_i above the target return are set to zero, *but these zeros are still included in the summation*. The calculation for Standard Deviation has no $\text{Min}()$ function.

Standard deviation is a measure of dispersion of data around its mean, *both above and below*. Target Downside Deviation is a measure of dispersion of data below some user selectable *target return* with all above target returns treated as underperformance of zero. Big difference.

Example Sortino Ratio Calculation

In this example, we will calculate the annual Sortino ratio for the following set of annual returns:

Annual Returns:
17%, 15%, 23%, -5%, 12%, 9%, 13%, -4%
Target Return: 0%

Although in this example we use a target return of 0%, any value may be selected, depending on the application, i.e. a futures trading system developer comparing different trading systems vs. a pension fund manager with a mandate to achieve 8% annual returns. Of course using a different target return will result in a different value for the Target Downside Deviation. If you are using the Sortino ratio to compare managers or trading systems, you should be consistent in using the same target return value.

First, calculate the numerator of the Sortino ratio, the average period return minus the target return:

$$\begin{aligned} & \text{Average Annual Return} - \text{Target Return:} \\ & (17\% + 15\% + 23\% - 5\% + 12\% + 9\% + 13\% - 4\%) \\ & \div 8 - 0\% = 10\% \end{aligned}$$

Next, calculate the Target Downside Deviation:

1. For each data point, calculate the difference between that data point and the target level. For data points above the target level, set the difference to 0%. The result of this step is the underperformance data set.

$$\begin{aligned} 17\% - 0\% &= 0\% \\ 15\% - 0\% &= 0\% \\ 23\% - 0\% &= 0\% \\ -5\% - 0\% &= -5\% \\ 12\% - 0\% &= 0\% \\ 9\% - 0\% &= 0\% \\ 13\% - 0\% &= 0\% \\ -4\% - 0\% &= -4\% \end{aligned}$$

2. Next, calculate the square of each value in the underperformance data set determined in the first step. Note that percentages need to be expressed as decimal values before squaring, i.e. 5% = 0.05.

$$\begin{aligned} 0\%^2 &= 0\% \\ 0\%^2 &= 0\% \\ 0\%^2 &= 0\% \\ -5\%^2 &= 0.25\% \\ 0\%^2 &= 0\% \\ 0\%^2 &= 0\% \\ 0\%^2 &= 0\% \\ -4\%^2 &= 0.16\% \end{aligned}$$

3. Then, calculate the average of all squared differences determined in Step 2. Notice that we do not “throw away” the 0% values.

$$\begin{aligned} & \text{Average: } (0\% + 0\% + 0\% + 0.25\% + \\ & 0\% + 0\% + 0\% + 0.16\%) \div 8 = 0.0513\% \end{aligned}$$

4. Then, take the square root of the average determined in Step 3. This is the Target Downside Deviation used in denominator of the Sortino Ratio. Again, percentages need to be expressed as decimal values before performing the square root function.

$$\begin{aligned} & \text{Target Downside Deviation:} \\ & \text{Square root of } \sqrt{0.0513\%} = 2.264\% \end{aligned}$$

Finally, we calculate the Sortino ratio:

$$\frac{10\%}{2.264\%} = 4.417 \text{ Sortino Ratio}$$

COMPARISON OF SORTINO RATIOS

WINTON	1.95
RED ROCK	1.78
LYNX	1.59
BLUETREND	1.56
TRANSTREND	1.42
MAN AHL	1.22
NEWEDGE CTA	1.11
ASPECT	0.87
GRAHAM	0.74
SUNRISE	0.55

The above table summarizes the Sortino ratios for a popular CTA index and a handful of well-known CTAs (from the September 2003 inception of Red Rock Capital's Systematic Global Macro program through March 2015). Graham is K4D-15V, Sunrise is Expanded Diversified, BlueTrend is from April 2004 and is now Systematica. Source: BarclayHedge. Past performance is not necessarily indicative of future performance.

Conclusion

The main reason we wrote this article is because in both literature and trading software packages, we have seen the Sortino ratio, and in particular the target downside deviation, calculated incorrectly more often than not. Most often,

we see the target downside deviation calculated by “throwing away all the positive returns and take the standard deviation of negative returns”. We hope that by reading this article, you can see how this is incorrect. Specifically:

- In Step 1 above, the difference with respect to the target level is calculated, unlike the standard deviation calculation where the difference is calculated with respect to the mean of all data points. If every data point equals the mean, then the standard deviation is zero, no matter what the mean is. Consider the following return stream: [-10, -10, -10, -10]. The standard deviation is 0, while the target downside deviation is 10 (assuming target return is 0).
- In Step 3 above, all above target returns are included in the averaging calculation. The above target returns set to 0% in step 1 are kept.
- The Sortino ratio takes into account both the frequency of below target returns as well as the magnitude of below target returns. Throwing away the zero underperformance data points removes the ratio's sensitivity to frequency of underperformance. Consider the following underperformance return streams: [0, 0, 0, -10] and [-10, -10, -10, -10]. Throwing away the zero underperformance data points results in the same target downside deviation for both return streams, but clearly the first return stream has much less downside risk than the second.

In this paper we presented the definition of the Sortino ratio and the correct way to calculate it. While the Sortino ratio addresses and corrects some of the weaknesses of the Sharpe ratio, neither statistic measures ongoing and future risks; they both measure the past “goodness” of a manager's or investment's return stream.

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About Red Rock Capital

Red Rock Capital is an award-winning commodity investment management firm. During the summer of 2015 Red Rock will proudly celebrate its 12th anniversary. The firm is led by Thomas Rollinger, who is most notably a devoted pupil and former protégé of quantitative hedge fund legend, Edward O. Thorp. Given recent developments with the firm, plus increasingly favorable market conditions, Red Rock is especially well-positioned to continue to grow and thrive in the managed futures industry.