Portfolio management with drawdown-based measures

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Abstract

This paper analyzes the portfolio management implications of using drawdown-based measures in allocation decisions. We introduce modified conditional expected drawdown (MCED), a new risk measure that is derived from portfolio drawdowns, or peak-to-trough losses, of demeaned constituents. We show that MCED exhibits the attractive properties of coherent risk measures that are present in conditional expected drawdown (CED) but lacking in the historical maximum drawdown (MDD) commonly used in the industry. This paper introduces a robust block bootstrap approach to calculating CED, MCED and marginal contributions from portfolio constituents. First, we show that MCED is less sensitive to sample error than CED and MDD. Second, we evaluate several drawdown-based minimum risk and equal-risk allocation approaches within the large scale simulation framework of Molyboga and L'Ahelec (2016) using a subset of hedge funds in the managed futures space that contains 613 live and 1,384 defunct funds over the 1993-2015 period. We find that the MCED-based equal-risk approach dominates the other drawdown-based techniques but does not consistently outperform the simple equal volatility-adjusted approach. This finding highlights the importance of carefully accounting for sample error, as reported in DeMiquel et al (2009), and cautions against over-relying on drawdown-based measures in portfolio management.

Institutional investors make investment decisions based on a variety of measures of risk and risk-adjusted performance with maximum historical drawdown, defined as the largest peak-to-valley loss, among the most popular measures. In fact, “Best practices in alternative investments: due diligence” (2010) require drawdown analysis as part of quantitative due diligence.

The purpose of this paper is to evaluate portfolio implications of using drawdowns in allocation decisions by examining several established and new drawdown-based approaches. We introduce a new drawdown-based measure, Modified Conditional Expected Drawdown (MCED), and demonstrate its attractive characteristics, suggest a robust block bootstrap approach for its calculation, and investigate the portfolio implications of utilizing this measure. We evaluate MCED using the large-scale simulation framework and realistic constraints of Molyboga and L’Ahelec (2016) imposed on a subset of hedge funds in the managed futures space that contains 613 live and 1,384 defunct funds over the 1993-2015 period. We find that the MCED-based equal-risk approach dominates the other drawdown-based techniques while underperforming the equal volatility-adjusted approach.

Goldberg and Mahmoud (2014) formalize drawdown risk as Conditional Expected Drawdown (CED), the tail mean of maximum drawdown distributions, show that CED is a coherent measure as defined in Artzner et al (1999) and investigate two portfolio construction approaches that either minimize CED or equalize constituent contributions to portfolio CED. The former approach represents a minimum risk approach while the latter is a variation of risk-parity introduced in Qian (2006). The historical performance of portfolio constituents is factored into the CED calculation through the constituents’ total cumulative performance, or the slope of the cumulative return function, and their contribution to portfolio performance during bad periods. While contribution to portfolio performance is likely to detect diversifying portfolio constituents, incorporating total cumulative performance reflects performance chasing and is likely to have negative implications. We modify Conditional Expected Drawdown to capture its diversifying characteristics while eliminating performance chasing aspects and refer to the new risk measure as Modified Conditional Expected Drawdown (MCED). This modification eliminates the slopes of individual portfolio constituents by demeaning returns while also preserving all the attractive properties of Conditional Expected Drawdown documented in Goldberg and Mahmoud (2014). We demonstrate that MCED is less sensitive to sample error than CED and MDD, using 1,000 simulations with 5 hypothetical portfolio constituents each possessing 36 months of track record. The standard deviation of errors of the MCED approach is 12.2%, lower than the 17.39% of the CED approach and the 17.07% of the MDD approach.

Chekhlov et al (2005) introduce a family of risk measures called Conditional Drawdown (CDD), the tail mean of drawdown distributions, investigate its mathematical characteristics and discuss applications of the new measure to asset allocation decisions. They also suggest a block bootstrap procedure for the calculation of CDD and show that the procedure is robust after approximately 100 simulations. The bootstrap approach is used because analytic solutions are not feasible and, as shown in Douady et al (2000), even the calculation of expected drawdown for standard Brownian motion can be very complex. Block bootstrap is particularly attractive because it preserves the serial and cross-correlational characteristics of the original dataset, likely making it superior to the rolling historical maximum drawdown algorithm of Goldberg and

Mahmoud (2014) that was used for illustrative purposes. We apply a similar block bootstrap procedure to CED and MCED calculations, using 200 simulations for robustness.

Drawdown-based measures can be applied to portfolios of traditional and alternative investments such as stocks, bonds, real estate and hedge funds. In this paper, we evaluate MCED within the large-scale simulation framework of Molyboga and L'Ahelec (2016) imposed on a subset of hedge funds in the managed futures space that contains 613 live and 1,384 defunct funds over the 1993-2015 period. The framework incorporates the standard requirements of institutional investors regarding track record length, the amount of assets under management (AUM), and the number of funds in the portfolio. The methodology closely mimics the portfolio management decisions of institutional investors and incorporates investment constraints and investor preferences to produce results that are relevant for investors[3].

Following Molyboga and L'Ahelec (2016), we evaluate out-of-sample performance with several commonly used measures of standalone performance and marginal portfolio contribution[4]. Annualized return, Sharpe and Calmar[5] ratios and maximum drawdown are used to measure standalone performance. We evaluate marginal portfolio contribution by measuring the improvement in Sharpe and Calmar ratios that results from replacing a modest 10% allocation to the investor's original portfolio with a 10% allocation to a simulated CTA portfolio. In this paper, we use the standard 60-40 portfolio of stocks and bonds with monthly

[3] The framework of Molyboga and L'Ahelec (2016) is customizable to the preferences and constraints of individual investors such as investment objectives, number of funds in the portfolio, and rebalancing frequency.
[4] The framework is flexible and can incorporate customized performance measures selected by the investor.
[5] Calmar ratio is defined as the ratio of the annualized excess return to the maximum historical drawdown.

returns between January 1999 and June 2015 as the investor's original portfolio. Though this portfolio is often used in the literature, the framework is flexible to the choice of investor benchmark.

We find that a modest 10% allocation to CTA portfolios improves the performance of the original 60-40 portfolio of stocks and bonds for all portfolio construction methodologies considered in the study. For the out-of-sample period between January 1999 and June 2015, a 10% allocation to managed futures improves the Sharpe ratio of the original portfolio from 0.365 to 0.390-0.404, on average, depending on the portfolio construction methodology. The MCED-based equal-risk approach results in a Sharpe ratio of 0.400. Similarly, the Calmar ratio improves from 0.088 to 0.097-0.104, on average, with the MCED-based equal-risk approach delivering an average Calmar ratio of 0.101. Blended portfolios have higher Sharpe ratios in at least 89.9% of simulations and higher Calmar ratios in at least 91.9% of simulations. We find that on a standalone basis the MCED-based equal-risk approach dominates the other drawdown-based techniques but does not outperform the equal volatility-adjusted approach (EVA) highlighted in Molyboga and L'Ahelec (2016). For the out-of-sample period between January 1999 and June 2015, the MCED-based equal-risk portfolio delivered an average Sharpe ratio of 0.339, which is higher than the 0.286-0.296 achieved by the other drawdown-based approaches, the 0.308 of random portfolios, and the 0.326 of a naïve 1/N approach but slightly lower than the 0.347 of the equal volatility-adjusted (EVA) method. Calculating the Calmar ratio produces similar relative results with the MCED-based risk-parity equal-risk approach delivering an average Calmar ratio of 0.163, which is higher than the ratios of the other approaches except EVA, which delivers an average Calmar ratio of 0.166.

The remainder of the paper is organized as follows: Section I discusses several drawdown-based measures and introduces MCED; Section II presents the block bootstrap methodology for calculating MCED and CED; Section III describes the CTA data and accounts for biases within the data; Section IV describes the simulation framework; Section V presents empirical out-of-sample results; and Section VI concludes.

I. Drawdown-based measures

In this section, we describe several measures of drawdown that are either commonly used in the industry or documented in academic literature. Then we introduce the Modified Conditional Expected Drawdown (MCED) and discuss its characteristics.

Institutional investors use a variety of measures of risk with the historical maximum drawdown (MDD), defined as the largest peak-to-valley loss, among the most popular measures. "Best practices in alternative investments: due diligence" (2010) require drawdown analysis as an important part of quantitative due diligence. Therefore, there may be very significant value in constructing portfolios with low drawdowns. However, minimizing maximum historical drawdowns has several known issues. First, the issue of overfitting – excessive optimization for noise in past returns without accounting for potential alternative outcomes - typically results in poor out-of-sample performance as documented in DeMiquel et al (2009) for mean-variance optimization approaches. Second, the numerical calculation of MDD can be quite involved. Finally, focusing on maximum historical drawdown, the worst case scenario from within the drawdown distribution, can potentially result in sub-optimal behavior.

Chekhlov et al (2005) and Goldberg and Mahmoud (2014) suggest looking beyond a single point of historical maximum drawdown and introduce drawdown-based measures that are based on the left tail of the drawdown distribution. In this paper, we consider Conditional Expected Drawdown (CED) introduced in Goldberg and Mahmoud (2014) and defined as the tail mean of maximum drawdown distributions

$$\mathrm{CED}_{\alpha}(\xi) = \mathrm{E}\left(\mu(\xi) \mid \mu(\xi) < DT_{\alpha}(\xi)\right) ,$$

where $\mu(\xi)$ represents the maximum drawdown distribution of random variable ξ [6] and $DT_{\alpha}(\xi)$ is the quantile of the maximum drawdown distribution that corresponds to probability α.

This measure resembles expected shortfall, or CVaR, but uses the distribution of maximum drawdown instead of the distribution of returns. Similar to expected shortfall, CED is a coherent risk measure that possesses a number of attractive characteristics described in Artzner et al (1999). It is also a homogeneous function of order one, which makes it easy to calculate the contribution to portfolio CED from its constituents using the Euler equation.

Let us denote the demeaned portfolio component $\tilde{\xi}_i = \xi_{\mathrm{i}} - \bar{\xi}_{\mathrm{i}}$, where $\bar{\xi}_{\mathrm{i}}$ is the sample mean of portfolio component i. The demeaned portfolio is, therefore, $\tilde{\xi} = \mathrm{w}_1\tilde{\xi}_1 + \cdots + \mathrm{w}_{\mathrm{n}}\tilde{\xi}_{\mathrm{n}}$.

We define the modified conditional expected drawdown as follows:

$$\mathrm{MCED}_{\alpha}(\xi) = \mathrm{E}\left(\mu(\tilde{\xi}) \mid \mu(\tilde{\xi}) < DT_{\alpha}(\tilde{\xi})\right) .$$

[6] Portfolio return is a weighted average of its components' returns $\xi = \mathrm{w}_1\xi_1 + \cdots + \mathrm{w}_{\mathrm{n}}\xi_{\mathrm{n}}$ such that $\mathrm{w}_1 + \cdots + \mathrm{w}_{\mathrm{n}} = 1$. In this study, portfolio weights are non-negative because manager allocations cannot be negative, but the MCED can be applied to long-short portfolios that have both positive and negative weights.

By construction, MCED has the same properties of coherence and positive homogeneity as CED but, as we show in the next section, MCED is less sensitive to sample error than CED or MDD. Moreover, we present a methodology for generating the maximum drawdown distribution μ using the historical performance of the portfolio constituents and their weights.

II. Simulation-based calculation of CED and MCED

In this section, we describe previously documented approaches for calculating CED and introduce a block bootstrap methodology for estimating the maximum drawdown distribution that is used in the calculation of CED and MCED. Bootstrap methodologies, introduced in Efron (1979) and Efron and Gong (1983), are commonly used in statistics to estimate distributions when analytic solutions are unsuitable or difficult to obtain. An analytic solution for the drawdown distribution is not feasible for realistic time-series with serial correlation and fat tails. Even the relatively simple case of standard Brownian motion involves a very complex derivation of expected drawdown, as documented in Douady et al (2000).

The block bootstrap technique is particularly attractive because it preserves the serial and cross-correlational characteristics of the original dataset[7]. The bootstrap procedure is likely superior to the rolling historical maximum drawdown algorithm of Goldberg and Mahmoud (2005) that was used for illustrative purposes because rolling historical maximum drawdown relies heavily on a single path and uses overlapping observations that are likely to include the same maximum drawdown multiple times. We apply a similar block bootstrap procedure to the

[7] Fama and French (2010) apply a bootstrap methodology that uses single month cross-sections of data. Blocher, Cooper and Molyboga (2015) utilize a block bootstrap technique for Commodity Trading Advisors that specialize in trading commodities.

CED and MCED calculations and show that, unlike CED and MDD, MCED is robust to sample error. Finally, we demonstrate an efficient approach to calculating marginal contribution to CED and MCED based on the algorithm of Goldberg and Mahmoud (2014).

A. Block bootstrapping methodology

Chekhlov et al (2005) suggest a block bootstrap procedure for the calculation of drawdown-based measures and demonstrate that the procedure is robust after approximately 100 simulations. Block bootstrap is a technique that applies multiple samplings with replacement to blocks of cross-sections of the original dataset to produce realistic "what if" scenarios. Each scenario has the same number of constituents and the same length of time-series as the original dataset. We generate 200 simulations to calculate drawdown-based measures to ensure that calculations are robust. Simulations are used to calculate potential distributions of MDD, CED and MCED for any non-negative portfolio weights such that $\mathrm{w}_1 + \cdots + \mathrm{w}_\mathrm{n} = 1$ using simulated portfolio returns $\xi = \mathrm{w}_1\xi_1 + \cdots + \mathrm{w}_\mathrm{n}\xi_\mathrm{n}$. Drawdown contributions from individual components are calculated as described in the next sub-section.

B. Algorithm for calculating contribution to portfolio MCED and CED

Following Goldberg and Mahmoud (2014), we denote marginal contribution to portfolio risk measure ρ as the derivative of the risk measure with respect to its component i:

$$MRC_i^{\rho}(\xi) = \frac{\partial \rho(\xi)}{\partial w_i}.$$

For any homogeneous function of order one, such as MCED and CED, the portfolio risk can be decomposed using Euler's formula:

$$\rho(\xi) = \sum_{i=1}^{n} w_i MRC_i^{\rho}(\xi) = \sum_{i=1}^{n} TRC_i^{\rho}(\xi),$$

where $TRC_i^{\rho}(\xi) = w_i MRC_i^{\rho}(\xi)$ is the total risk contribution from component i. Equal-risk approaches, such as classic risk parity, use weights that result in an equal total return contribution from each component.

Goldberg and Mahmoud (2014) show that

$$MRC_i^{CED_\alpha}(\xi) = E\left[\left(\xi_{i,t_K} - \xi_{i,t_J}\right) \mid \mu(\xi) < DT_\alpha(\xi)\right],$$

where the maximum drawdown of the portfolio with cumulative return path of $\{\xi^t, t = 1, \cdots, T\}$ is achieved between time t_K and t_J with

$$\mu(\xi) = \mu\left(\xi^t, t = 1, \cdots, T\right) = \min_{1 \le j < n} \min_{j < k \le n} \left(\xi^{t_k} - \xi^{t_j}\right) = \xi^{t_K} - \xi^{t_J} .$$

This formulation simplifies the calculation of marginal contribution to CED and MCED for datasets generated using the block bootstrap methodology.

C. Sensitivity to sampling error

We investigate the sensitivity of MDD, CED and MCED to sampling error by generating 1,000 random scenarios, calculating weights that minimize MDD, CED and MCED for each and evaluating them against the true optimal weights. Each scenario uses 5 uncorrelated assets with 36 monthly returns that are independent and identically distributed and follow a standard normal N(0,1) distribution. By construction, true optimal weights are equal to 20% but the introduction of sampling error results in differing sets of weights. We calculate the average distance between the calculated optimal weights and the true optimal weights to evaluate the sensitivity of the measures to sampling error.

Figure 1 presents the density functions of the sample errors of the three methodologies.

<Figure 1>

Table I reports the results of the sensitivity testing. The table presents the ranges of sample errors, the percentages of errors between -20% and +20%, and the standard deviations of errors.

<Table I>

The range of MCED errors is 64.71%, whereas both CED and MDD have ranges of 100%. The percentage of relatively small errors between -20% and +20% is 94.14% for MCED, which is higher than the 86.90% for CED and the 86.56% for MDD. The standard deviation of its errors is 12.20%, which is lower than the 17.39% for CED and the 17.07% for MDD. Each metrics shows that MCED is the most robust of the three drawdown-based measures to sample error.

III. Data

In this study, we use the BarclayHedge database, the largest publicly available database of Commodity Trading Advisors with 989 active and 3,784 defunct funds between December 1993 and June 2015[8]. Appendix A outlines the standard data processing procedures used to address biases in the data and limit the scope of the study to the funds that are relevant for institutional investors who make direct investments. We include the graveyard database that contains defunct funds to account for survivorship bias. We also explicitly account for backfill and incubation biases that arise due to the voluntary nature of self-reporting in CTA and hedge fund databases[9]. We combine two standard methodologies to mitigate these biases. The first

[8] Joevaara, Kosowski and Tolonen (2012) report that the BarclayHedge database provides the highest quality data from among CISDM (Morningstar), Lipper (formerly TASS), Eurekahedge and BarclayHedge.

[9] Fund managers start reporting to a CTA database to raise capital from outside investors only if the track record, generated using proprietary capital during the "incubation" period, is attractive. Then they typically "backfill" the

methodology, introduced in Fama and French (2010), limits the tests to those funds that managed at least US $10 million normalized to December 2014 values. Since a significant portion of CTAs reported only net returns for an extended period of time prior to their initial inclusion of AUM data, utilizing the Fama and French (2010) technique exclusively would eliminate a significant portion of the dataset. To include these data, we apply the methodology of Kosowski, Naik and Teo (2007), which eliminates only the first 24 months of data for such funds. We further incorporate a liquidation bias of 1% as suggested in Ackermann, McEnally and Ravenscraft (1999). After accounting for the biases, our dataset includes 613 live and 1,384 defunct funds for the period between December 1995 and June 2015.

This study uses the Fung-Hsieh five factor model of primitive trend following systems, documented in Fung and Hsieh (2001), as benchmarks for measuring the performance of CTA portfolios. The factors include PTFSBD (bonds), PTFSFX (foreign exchange), PTFSCOM (commodities), PTFSIR (interest rates) and PTFSSTK (stocks). The 3-month secondary market rate Treasury bill series with ID TB3MS from the Board of Governors of the Federal Reserve System serves as a proxy for the risk-free rate. Table II reports summary statistics and tests of normality, heteroscedasticity and serial correlation in CTA returns by strategy and status.

<Table II>

The 60-40 portfolio of stocks and bonds is used as the benchmark portfolio following Anson (2011) who suggested the portfolio represents a typical starting point for a U.S. institutional investor. This blend is constructed using the S&P 500 Total Return Index and the JPM Global

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returns over the incubation period. Since funds with poor performance are unlikely to report returns to the database, incubation/backfill bias arises.

Government Bond Index. Table III presents the annualized excess return, standard deviation, maximum drawdown, Sharpe ratio and Calmar ratio of the 60-40 portfolio for the period between January 1999 and June 2015. Over this time period, the portfolio delivered a Sharpe ratio of 0.365 and a Calmar ratio of 0.088.

<Table III>

Figure 2 shows the performance of the portfolio from January 1999 to June 2015.

< Figure 2>

IV. Methodology

In this section, we describe the portfolio construction approaches evaluated in this study and the large-scale simulation framework employed.

A. Review of portfolio construction approaches considered in the study.

In this paper, we evaluate five drawdown-based approaches. Three of them are minimum risk portfolios: minimum MDD (MinMDD), minimum CED (MinCED) and minimum MCED (minMCED). Two additional drawdown-based approaches are the equal-risk approaches, RP_CED and RP_MCED, which use portfolio weights that result in equal total contribution to risk from each component. We use three benchmark portfolio construction approaches, including an equal notional (EN) approach, which is a naïve diversification 1/N method highlighted in DeMiquel, Garlappi and Uppal (2009), an equal volatility-adjusted (EVA) approach documented in Hallerbach (2012) and a random portfolio selection approach (Random). The approaches are evaluated using a large-scale simulation framework with realistic constraints[10].

B. Large-scale simulation framework.

This study uses the large scale simulation framework introduced in Molyboga and L'Ahelec (2016) with 1,000 simulations for the out-of-sample period between January 1999 and June 2015. The methodology mimics portfolio management decisions of institutional investors who rebalance portfolios at the end of each month. The first allocation decision is made in December 1998. Due to the delay in CTA reporting, documented in Molyboga, Baek and Bilson (2015), the investor has return information available only through November 1998 at the time of decision making. Therefore, the investor considers all funds that have a complete set of monthly returns between December 1995 and November 1998. Following Molyboga and L'Ahelec (2016), the investor excludes all funds in the bottom quintile of AUM among the funds considered[11]. Then the investor randomly chooses 5 funds from the remaining pool of CTAs and allocates to them using the five drawdown-based approaches and the three benchmark methods. Monthly returns are recorded for each portfolio construction approach for January 1999 using the liquidation bias adjustment for funds that liquidate during the month. At the end of January 1999, the pool of CTAs is updated and constituents of the original portfolio not included in the new CTA pool, either due to liquidation or decrease in relative AUM level, are

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[10] See Appendix B for technical definitions of the allocation approaches.

[11] Molyboga, Baek and Bilson (2015) argue that this relative AUM threshold is more appropriate than the fixed AUM approach commonly used in the literature because the average level of AUM has increased substantially over the last 20 years.

randomly replaced with funds from the new pool. Each portfolio is then rebalanced again using the original portfolio construction methodologies. The process is repeated until the end of the out-of-sample period in June 2015. A single simulation results in six out-of-sample return streams between January 1999 and December 2015 – one for each of the portfolio construction approaches. Finally, performance results are evaluated based on out-of-sample results across 1,000 simulations.

Table IV reports the average AUM threshold level for each year and the average number of funds meeting that threshold. The AUM threshold represents the 20th percentile of AUM among all active fund managers with a track record of at least 36 months.

<Table IV>

Variation in the AUM threshold and the number of active funds over time reflects industry dynamics, which are driven primarily by recent performance and industry maturity.

C. *Evaluation of out-of-sample results*

We evaluate out-of-sample performance using Sharpe and Calmar ratios as standalone performance measures[12] as well as portfolio contribution metrics. Performance contribution is calculated as the resultant difference in Sharpe ratio and Calmar ratio from replacing 10% of the original benchmark portfolio with portfolios of CTA funds constructed within the simulation framework. In this paper, we use the standard 60-40 portfolio of stocks and bonds with monthly returns between January 1999 and June 2015.

[12] Molyboga and L'Ahelec (2016) utilize several additional measures of performance to demonstrate the flexibility of the framework. This study uses only two of the measures for brevity.

V. Empirical results

In this section, we present empirical out-of-sample results obtained within the large-scale simulation framework with realistic constraints. We demonstrate that the MCED-based equal-risk portfolio is superior to the other drawdown-based methods considered in the study but fails to outperform a volatility-based equal risk approach. We also show that the marginal benefit of CTA portfolios to the traditional 60-40 portfolio of stocks and bonds is positive and robust across all performance measures and portfolio methodologies considered in the study.

A. Analysis of out-of-sample performance of CTA portfolios as standalone investments

We evaluate out-of-sample performance using means and medians of the distributions generated using the large-scale simulation framework. A boostrapping methodology[13] is used to draw statistical inference because simulations are not independent and, thus, classic statistics is not appropriate.

Table V reports means and medians of the distributions of returns, volatilities, Sharpe and Calmar ratios and maximum historical drawdowns for each portfolio construction approach.

<Table V>

[13] Appendix C describes the bootstrapping procedure that is used to draw statistical inference about relative performance of the portfolio construction approaches and estimation of p-values.

The superscript star indicates that the performance measure of a given portfolio approach exceeds that of the RANDOM portfolio at the 99% confidence level. The subscript star shows that the performance measure of a given portfolio approach is lower than that of the RANDOM portfolio at the 99% confidence level. Most drawdown-based approaches except RP_MCED, the MCED-based equal-risk approach, produce Sharpe ratios that are inferior to the 0.308 of the RANDOM methodology. Bootstrapping suggests that the lower Sharpe ratios are statically different at the 99% level for MinMDD, MinCED and RP_CED and at the 98% level for the MinMCED. By contrast, EN, EVA and RP_MCED outperform random portfolios at the 99% confidence level with the equal volatility-adjusted approach leading with an average Sharpe ratio of 0.347, followed by the MCED-based equal-risk approach with an average Sharpe ratio of 0.339 and the naïve 1/N approach with an average Sharpe ratio of 0.326. This finding is consistent with DeMiquel, Garlappi and Uppal (2009), which documents the superior out-of-sample performance of the naïve 1/N (EN) approach[14], and Molyboga and L'Ahelec (2016), which documents the superior performance of equal-risk approaches relative to random and minimum risk methodologies. Median values reported in Panel B demonstrate consistent results.

The large scale simulation framework produces distributions of out-of-sample performance that can be visualized using standard box and whisker plots to provide additional

[14] DeMiquel, Garlappi and Uppal (2009) also point out that a naïve 1/N approach should dominate a random portfolio in terms of Sharpe ratio due to the concavity of the Sharpe ratio. Jensen's inequality states that $Eg(X) \leq g(Ex)$ for any concave function g such as the Sharpe ratio. See Rudin (1986) for a detailed explanation of Jensen's inequality.

insights[15]. Figure 3 displays the distributions of Sharpe ratios for each portfolio construction approach.

<Figure 3>

The breadth of each distribution represents the role of chance and highlights the importance of using a large-scale simulation framework to evaluate portfolio techniques, as discussed in detail in Molyboga and L'Ahelec (2016). For example, minimum risk portfolios tend to have wider distributions of outcomes with large negative outliers – something that risk-averse investors may want to consider when they make their investment decisions[16].

Figure 4 displays the distributions of Calmar ratios for each portfolio approach.

<Figure 4>

Although the choice of a portfolio construction methodology based on the distribution of outcomes ultimately depends on the preferences of a specific investor, Figure 4 indicates that the role of chance is significant and the EVA and RP_MCED methodologies look more attractive than any of the traditional minimum drawdown methodologies.

B. Evaluation of portfolio contribution

We further analyze the marginal contribution of the portfolio construction approaches to a traditional 60-40 portfolio of stocks and bonds by comparing the performance of blended portfolios that replace a modest 10% allocation to the benchmark portfolio with 10% to the

[15] The box contains the middle two quartiles, the thick line inside the box represents the median of the distribution and the whiskers are displayed at the top and bottom 5 percent of the distribution.

[16] Molyboga, Baek and Bilson (2015) utilize stochastic dominance and utility functions to compare distributions. Both approaches capture the risk-aversion characteristics in investors' preferences and can account for the differences in distributions shown in Figure 3.

hypothetical CTA portfolios[17]. Table VI reports the Sharpe and Calmar ratios of the blended portfolios and the percentage of scenarios that result in blended portfolios having higher Sharpe and Calmar ratios than the original 60-40 portfolio. Panel A reports mean results for the Sharpe and Calmar ratios. Panel B presents median values.

Table VI

The marginal benefit of managed futures is very robust across portfolio construction approaches and simulations. A modest allocation to hypothetical CTA portfolios improves the performance of the original 60-40 portfolio of stocks and bonds for all portfolio construction methodologies considered in the study. For the out-of-sample period between January 1999 and June 2015, the 0.365 Sharpe ratio of the original portfolio increases to between 0.390 and 0.404, on average, depending on the portfolio construction methodology. The MCED-based equal-risk approach yields a Sharpe ratio of 0.40. Similarly, the Calmar ratio improves from 0.088 to 0.0970-0.104, on average, with the MCED-based equal-risk approach delivering an average Calmar ratio of 0.101. The improvement in the Sharpe ratio is observed across at least 89.9% of simulations and the improvement in the Calmar ratio is even more robust with at least 91.9% of simulations resulting in superior Calmar ratios for the blended portfolios versus the original portfolio.

The results are striking but consistent with the academic literature on managed futures[18]. Our findings are consistent with Kat (2004), Lintner (1996), Abrams, Bhaduri and

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[17] Molyboga and L'Ahelec (2016) argue that marginal contribution analysis is particularly important for investors with exposure to a large number of systematic sources of return.

[18] It is important to note that relative performance results depend on the evaluation window as described in Anderson, Bianchi and Goldberg (2012). Therefore, sub-period analysis can provide additional insights into key factors such as market environment that can potentially impact relative performance of strategies.

Flores (2009), and Chen, O'Neill and Zhu (2005) that report a positive contribution of managed futures to traditional portfolios on average. Molyboga and L'Ahelec (2016) demonstrate that the positive contribution of managed futures to a traditional 60-40 portfolio of stocks and bonds is robust across all risk-based approaches considered in their study.

We investigate the impact of the size of allocation to managed futures on the original 60-40 portfolio by repeating the above analysis for a range of allocations to managed futures from 5% to 60%. Table VII reports the percentage of scenarios with higher Sharpe ratios in Panel A, mean Sharpe ratios in Panel B and median Sharpe ratios in Panel C.

Table VII

Table VII shows that mean Sharpe ratios increase until the CTA allocation reaches 40-50% and decline thereafter. The MCED-based equal-risk approach yields a Sharpe ratio of 0.382 for a blended portfolio with a 5% allocation to CTA portfolios and a Sharpe ratio of 0.503 for a portfolio with a 50% allocation to CTA portfolios. However, that improvement in average performance comes with higher risk. For example, the MCED-based equal-risk approach improves the Sharpe ratio of the investor portfolio in 97.6% of scenarios for a 5% allocation to CTA portfolios and 86.6% of scenarios for the 50% allocation that yields the highest average Sharpe.

Figure 5

Figure 5 displays distributions of the Sharpe ratio of blended portfolios utilizing the MCED-based equal-risk methodology with an allocation to CTA portfolios ranging between 5% and 60%. As discussed above, the choice of the appropriate allocation for an investor is determined by his or her specific preferences.

We further investigate the impact of the size of allocation to managed futures on the original 60-40 portfolio by evaluating the Calmar ratios of blended portfolios. Table VIII reports the percentage of scenarios with higher Calmar ratios in Panel A, mean Calmar ratios in Panel B and median Calmar ratios in Panel C.

Table VIII

A comparison of Panel A in Table VII with Panel A in Table VIII suggests that the improvement in the Calmar ratio from an allocation to CTA portfolios is more robust across scenarios than is the improvement in the Sharpe ratio. Moreover, while the Sharpe ratio reaches its maximum at a 40%-50% allocation to CTA portfolios, the Calmar ratio continues to increase monotonically over the whole range between 5% and 60% for each portfolio methodology. This suggests that the potential of managed futures to reduce drawdowns of the original 60-40 portfolio goes beyond more volatility reduction due to the lead-lag cross-correlation structure that is complementary beyond contemporaneous correlations.

The MCED-based equal-risk approach yields a Calmar ratio of 0.094 for a blended portfolio with a 5% allocation to CTA portfolios and a Calmar ratio of 0.227 for a portfolio with a 60% allocation to CTA portfolios. As in the case of the Sharpe ratio, the improvement in average performance comes with higher risk. The MCED-based equal-risk approach improves the Calmar ratio of the investor portfolio in 99.2% of scenarios with a 5% allocation to CTA portfolios and in 94.8% of scenarios with a 60% allocation. The MCED-based equal-risk approach outperforms the EVA over the whole range of allocations to CTA portfolios on average, as measured by mean and median.

Figure 6

Figure 6 displays distributions of the Calmar ratios of blended portfolios based on the MCED-based equal-risk methodology with an allocation to CTA portfolios ranging between 5% and 60%. While the average Calmar ratio increases monotonically over the whole range of allocations to CTA portfolios between 5% and 60%, the breadth of the distribution and the size of the left tail also increase.

VI. Concluding remarks

This paper evaluates portfolio management implications of using drawdown-based measures in allocation decisions. We introduce modified conditional expected drawdown (MCED) and show that MCED exhibits the attractive properties of coherent risk measures present in conditional expected drawdown (CED) but lacking in the historical maximum drawdown (MDD) commonly used in the industry. We introduce a robust block bootstrap approach that uses historical performance to calculate MCED and the marginal contributions to MCED from portfolio constituents. MCED is less sensitive to sample error than are either CED or MDD, which makes it more attractive as an implementable portfolio management methodology.

We examine several drawdown-based minimum risk and equal-risk approaches within the large scale simulation framework of Molyboga and L'Ahelec (2016) using a subset of hedge funds in the managed futures space that contains 613 live and 1,384 defunct funds over the 1993-2015 period. We find that CTA investments make a significant portfolio contribution to a

60-40 portfolio of stocks and bonds over the 1999-2015 out-of-sample period. This result is robust across portfolio methodologies and performance measures.

We find that the MCED-based equal-risk approach dominates the other drawdown-based techniques but does not consistently outperform the simple equal volatility-adjusted approach. This finding highlights the importance of carefully accounting for sample error, as reported in DeMiquel et al (2009), and cautions against over-relying on drawdown-based measures in portfolio management.

Appendix A. Data Cleaning.

Since the paper focuses on the evaluation of direct fund investments, we excluded all funds from the BarclayHedge database that are multi-advisors or report returns gross-of-fees. These exclusions reduced the fund universe to 4,773 funds with 989 active and 3,784 defunct funds for the period between December 1993 and June 2015. Then we performed a few additional data filtering procedures to account for biases and potential errors in the data and produce results that are relevant for institutional investors. First, we eliminated all null returns at the end of the track records of defunct funds which is a typical reporting issue inherent within hedge fund databases. Then we excluded managers with less than 24 months of data which limited the data set to 3,321 funds. Additionally, we eliminated all funds with maximum assets under management of less than US $10 million which further limited the data set to 2,009 funds. Finally, we excluded funds with one or more monthly returns in excess of 100% which resulted in the final pool of 1,997 funds, 613 of which were active and 1,384 of which were defunct.

Appendix B. Allocation approaches.

In this study, we consider three minimum drawdown and several equal-risk approaches. They include equal notional (EN), equal volatility-adjusted (EVA), minimum drawdown (MDD), minimum Conditional Expected Drawdown (CED), minimum Modified Conditional Expected Drawdown (MCED), CED-based equal-risk portfolio (RP_CED) and MCED-based equal-risk portfolio (RP_MCED).

1) Equal notional (EN) allocation is a simple equal weighting (or naïve diversification) approach:

$$w_i = 1/N$$

where *N* is the number of funds in the portfolio and w_i is the weight of fund i.

2) Equal volatility-adjusted (EVA) allocation is similar to the equal notional approach with exposure to each fund adjusted for the fund's volatility which is estimated using the standard deviation of its in-sample excess returns:

$$w_i = \frac{1/\sigma_i}{\sum_{j=1}^{N}\left[1/\sigma_j\right]}$$

3) The MinMDD approach produces an allocation with the minimum historical drawdown.

4) The MinCED approach produces an allocation with the minimum CED for a confidence level of 90%.

5) The MinMCED approach produces an allocation with the minimum MCED for a confidence level of 90%.

6) The RP_CED is the solution to the following optimization problem:

$$\min_{w} \sum_{i=1}^{N} \left(\frac{\partial \rho}{\partial w_i} \frac{w_i}{\rho} - \frac{1}{N} \right)^2$$

such that $\sum_{i=1}^{N} w_i = 1$ and $w_i \geq 0$ and $\rho = CED$.

7) RP_MCED is the solution to the following optimization problem:

$$\min_{w} \sum_{i=1}^{N} \left(\frac{\partial \rho}{\partial w_i} \frac{w_i}{\rho} - \frac{1}{N} \right)^2$$

such that $\sum_{i=1}^{N} w_i = 1$ and $w_i \geq 0$ and $\rho = MCED$.

8) A random portfolio (RANDOM) is used as a benchmark approach for portfolio allocation. First, a random number x_i between 0 and 1 is generated. Then random portfolio weights are normalized by setting $w_i = \frac{x_i}{\sum_{j=1}^{N} x_j}$.

Appendix C. Bootstrapping procedure.

The bootstrapping procedure follows each step of the simulation framework but limits the set of portfolio construction approaches to the Random portfolio methodology against which we compare all other approaches[19]. Each simulation set consists of 1,000 simulations. The bootstrapping procedure includes 400 sets of simulations, a sufficient number to estimate p-

[19] The framework is flexible in comparing any two approaches to each other but requires performing additional bootstrapping simulations based on an investor's particular areas of interest.

values with high precision. A comparison of the performance metrics of the original simulation to the set of bootstrapped simulations gives the p-values reported in the empirical results section.

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Tables and Figures.

Table I. Sensitivity of minimum drawdown weights to sample error
This table presents results of sensitivity of weights that minimize MDD, CED and MCED to sample error. It is based on 1,000 simulations with 5 portfolio constituents. Error is defined as the difference between portfolio weights and the true optimal weight of 20%. The table reports the range of the error, percentage of error between -20% and 20%, and standard deviation of the errors for each approach.

Drawdown measure	Range	Percentage of errors between -20% and 20%	Standard deviation of errors
MDD	100.00%	86.56%	17.07%
CED	100.00%	86.90%	17.39%
MCED	64.71%	94.14%	12.20%

Table II. Summary statistics and tests of normality, heteroscekasticity and serial correlation in CTA returns

This table reports statistical properties of fund returns and residuals by strategy and current status. Columns one presents the number of funds in each category. Columns two and three report the cross-sectional mean of the Fung-Hsieh (2001) five-factor model monthly alpha and the t-statistic of alpha. Columns four and five display the cross-sectional mean of kurtosis and skewness of fund residuals. Column six reports the percentage of funds for which the null hypothesis of normal distribution is rejected by the Jarque-Bera test. Column seven reports the percentage of funds for which the null hypothesis of homoskedasticity is rejected by the Breusch Pagan test. Column eight reports the percentage of funds for which the null hopythesis of zero first-order autocorrelation is rejected by Ljung-Box test. All tests are applied to fund residuals and the p-value is set at 10% level.

		Mean				Test of normality	Test of hetero-skedasti-city	Test of auto-correlat-ion
	number of funds	alpha	t-stat of alpha	kurtosis	skewness	Funds with Jarque-Bera p<0.1	Funds with Breusch Pagan p<0.1	Funds with Jarque-Bera p<0.1
All Funds	1997	0.38%	0.84	4.23	0.09	39%	21%	20%
By Strategy:								
Arbitrage	24	0.05%	0.51	6.61	-0.12	50%	4%	13%
Discretionary	35	0.28%	0.65	4.30	0.29	29%	6%	6%
Fundamental - Agricultural	45	0.50%	0.75	5.79	0.34	51%	11%	31%
Fundamental - Currency	97	0.45%	0.90	4.41	0.21	47%	14%	22%
Fundamental - Diversified	112	0.41%	0.95	4.13	0.17	48%	21%	14%
Fundamental - Energy	25	0.27%	0.45	4.89	0.31	52%	4%	8%
Fundamental - Financial/Metals	81	0.20%	0.64	4.75	0.07	47%	12%	19%
Fundamental - Interest Rates	12	-0.12%	-0.30	3.17	-0.05	17%	50%	33%
Option Strategies	89	-0.09%	0.11	6.97	-0.43	78%	45%	25%
Stock Index	87	0.07%	0.59	4.14	0.07	38%	22%	13%
Stock Index,Option Strategies	3	-0.84%	-1.82	6.21	-1.03	67%	33%	33%
Systematic	41	0.34%	0.64	3.56	0.10	27%	15%	20%
Technical - Agricultural	10	-0.54%	-0.80	4.65	0.27	60%	20%	30%
Technical - Currency	204	0.35%	0.77	4.52	0.27	43%	13%	18%
Technical - Diversified	739	0.61%	1.13	3.76	0.07	35%	24%	22%
Technical - Energy	5	-0.10%	-0.10	3.42	0.12	20%	0%	20%
Technical - Financial/Metals	237	0.13%	0.68	3.66	0.02	24%	24%	18%
Technical - Interest Rates	12	0.33%	0.90	3.36	-0.09	25%	50%	25%
Other	139	0.34%	0.77	4.39	0.12	45%	22%	24%
By current status:								
Live funds	613	0.56%	1.32	4.19	0.07	41%	32%	24%
Dead funds	1384	0.29%	0.60	4.26	0.10	39%	16%	18%

Table III. Performance of a 60-40 portfolio of stocks and bonds for 1999-2015
This table reports annualized excess return, standard deviation, maximum drawdown, Sharpe ratio and Calmar ratio of the 60-40 portfolio of stocks and bonds for January 1999-June 2015. The portfolio is constructed using S&P 500 Total Return index and the JP Morgan Global Government Bond Index. 3-month Treasury bill (secondary market rate) is used as a proxy for the risk free rate. Calmar is the ratio of the annualized excess return and maximum drawdown.

Annualized Excess Return	3.47%
Annualized StDev	9.50%
Maximum Drawdown	39.29%
Sharpe ratio	0.365
Calmar ratio	0.088

Table IV. Annual statistics of Commodity Trading Advisors

This table presents threshold level of assets under management assigned at the bottom 20% level and the number of funds with at least 36 months of returns used in the study

Year	AUM threshold	Number of funds
1999	8,850,000	176
2000	5,600,000	180
2001	5,020,000	186
2002	5,037,700	194
2003	9,930,000	195
2004	10,874,500	220
2005	10,423,900	237
2006	13,348,000	248
2007	12,499,700	286
2008	11,734,200	314
2009	13,422,100	337
2010	13,970,300	354
2011	13,380,000	365
2012	10,290,700	354
2013	12,295,000	336
2014	11,527,300	315
2015	13,191,600	295

Table V. Mean and median statistics of out-of-sample performance 1999-2015

This table presents mean and median values of out-of-sample performance measures for each portfolio construction approach. Performance measures include annualized excess return, annualized excess standard deviation, Sharpe and Calmar ratio (defined as annualized excess return over maximum drawdown), and maximum drawdown. Panel A reports mean values, Panel B displays median values.

Panel A. Mean values

Portfolio Construction Approach	Return	Volatility	Sharpe	Calmar	Maximum Drawdown
RANDOM	3.56%	11.65%	0.308	0.151	28.27%
EN	3.54%	10.96%*	0.326*	0.158*	25.36%*
EVA	2.85%*	8.25%*	0.347*	0.166*	19.40%*
MinMDD	2.28%*	7.87%*	0.289*	0.125*	22.96%*
MinCED	2.27%*	7.87%*	0.286*	0.125*	22.83%*
RP_CED	3.16%*	10.93%*	0.289*	0.136*	27.23%*
MinMCED	2.23%*	7.28%*	0.296*	0.128*	20.84%*
RP_MCED	3.13%*	9.30%*	0.339*	0.163*	22.27%*

Panel B. Median values

Portfolio Construction Approach	Return	Volatility	Sharpe	Calmar	Maximum Drawdown
RANDOM	3.59%	11.44%	0.315	0.135	27.1%
EN	3.59%	10.77%*	0.327*	0.144*	24.15%*
EVA	2.81%*	8.01%*	0.345*	0.145*	18.23%*
MinMDD	2.16%*	7.61%*	0.294*	0.104*	21.40%*
MinCED	2.20%*	7.58%*	0.289*	0.103*	20.91%*
RP_CED	3.11%*	10.72%*	0.286*	0.122*	25.35%*
MinMCED	2.03%*	7.04%*	0.299*	0.110*	19.60%*
RP_MCED	3.09%*	9.26%*	0.339*	0.147*	21.35%*

Table VI. Portfolio contribution of CTA investments to the original investor portfolio 1999-2015

This table reports results of marginal contribution analysis. The original investor portfolio is represented by 60-40 portfolio of stocks and bonds. It has delivered Sharpe ratio of 0.365 and Calmar ratio of 0.088 over the period between January 1999 and June 2015. The first column presents Sharpe ratio of a blended portfolio that replaces 10% allocation of the original portfolio with 10% of the CTA portfolios constructed in the simulation framework. The second column reports Calmar ratio of blended portfolios. The third and fourth columns reports the percentage of time blended portfolios have higher Sharpe and Calmar ratios than then those of the original portfolio. Panel A reports mean values, Panel B displayes median values.

Panel A. Mean values

Portfolio Construction Approach	Sharpe	Calmar	Improvement in Sharpe	Improvement in Calmar
RANDOM	0.404	0.104	92.40%	97.20%
EN	0.404	0.104	98.10%*	99.60%*
EVA	0.397*	0.100*	98.70%*	99.20%*
MinMDD	0.391*	0.097*	89.90%*	92.10%*
MinCED	0.390*	0.098*	91.50%*	92.00%*
RP_CED	0.399*	0.102*	93.50%*	97.90%*
MinMCED	0.390*	0.097*	91.30%*	91.90%*
RP_MCED	0.400*	0.101*	97.40%*	99.20%*

Panel B. Median values

Portfolio Construction Approach	Sharpe	Calmar	Improvement in Sharpe	Improvement in Calmar
RANDOM	0.405	0.103	92.40%	97.20%
EN	0.404*	0.104*	98.10%*	99.60%*
EVA	0.397*	0.100*	98.70%*	99.20%*
MinMDD	0.390*	0.097*	89.90%*	92.10%*
MinCED	0.390*	0.098*	91.50%*	92.00%*
RP_CED	0.399*	0.101*	93.50%*	97.90%*
MinMCED	0.388*	0.096*	91.30%*	91.90%*
RP_MCED	0.400*	0.101*	97.40%*	99.20%*

Table VII. Sharpe ratios of blended portfolios

This table reports performance of blended portfolios for 1999-2015. Panel A reports percentage of scenarios with Sharpe ratio that exceeds Sharpe ratio of the investor's original portfolio. Panel B report cross-sectional mean of Sharpe ratios of blended portfolios. Panel C reports cross-sectional median of Sharpe ratios of blended portfolios.

Panel A. Percentage of scenarios with higher Sharpe	Allocation to CTA portfolios						
Portfolio Construction Approach	5%	10%	20%	30%	40%	50%	60%
RANDOM	93.6%	92.4%	91.4%	89.2%	85.7%	79.3%	71.6%
EN	98.1%	98.1%	97.4%	94.9%	91.1%	86.9%	79.6%
EVA	98.7%	98.7%	97.8%	96.8%	94.6%	90.8%	84.6%
MinMDD	90.5%	89.9%	88.5%	86.7%	83.9%	76.6%	71.1%
MinCED	91.7%	91.5%	87.6%	84.9%	81.2%	76.4%	68.8%
RP_CED	94.0%	93.5%	92.1%	90.1%	85.4%	79.5%	70.9%
MinMCED	91.5%	91.3%	90.3%	87.9%	84.3%	78.9%	73.8%
RP_MCED	97.6%	97.4%	96.2%	93.6%	90.3%	86.6%	81.8%

Panel B. Mean							
Portfolio Construction Approach	5%	10%	20%	30%	40%	50%	60%
RANDOM	0.384	0.404	0.441	0.469	0.480	0.473	0.449
EN	0.384	0.404	0.442	0.474	0.491	0.489	0.470
EVA	0.381	0.397	0.432	0.465	0.491	0.505	0.502
MinMDD	0.377	0.391	0.417	0.443	0.462	0.469	0.459
MinCED	0.377	0.390	0.417	0.442	0.461	0.468	0.458
RP_CED	0.382	0.399	0.432	0.457	0.469	0.462	0.438
MinMCED	0.377	0.390	0.417	0.444	0.466	0.477	0.472
RP_MCED	0.382	0.400	0.437	0.471	0.495	0.503	0.492

Panel C. Median							
Portfolio Construction Approach	5%	10%	20%	30%	40%	50%	60%
RANDOM	0.385	0.405	0.442	0.473	0.486	0.481	0.457
EN	0.384	0.404	0.444	0.476	0.495	0.494	0.474
EVA	0.380	0.396	0.430	0.461	0.487	0.501	0.498
MinMDD	0.377	0.390	0.415	0.440	0.458	0.469	0.460
MinCED	0.377	0.390	0.416	0.440	0.460	0.466	0.461
RP_CED	0.381	0.399	0.430	0.456	0.470	0.465	0.440
MinMCED	0.376	0.388	0.412	0.440	0.461	0.478	0.472
RP_MCED	0.382	0.400	0.435	0.469	0.491	0.496	0.488

Table VIII. Calmar ratios of blended portfolios

This table reports performance of blended portfolios for 1999-2015. Panel A reports percentage of scenarios with Calmar ratio that exceeds Calmar ratio of the investor's original portfolio. Panel B report cross-sectional mean of Calmar ratios of blended portfolios. Panel C reports cross-sectional median of Calmar ratios of blended portfolios.

Panel A. Percentage of scenarios with higher Calmar

	Allocation to CTA portfolios						
Portfolio Construction Approach	5%	10%	20%	30%	40%	50%	60%
RANDOM	97.2%	97.2%	96.3%	95.3%	92.9%	89.8%	88.9%
EN	99.6%	99.6%	99.3%	98.8%	97.8%	96.4%	94.3%
EVA	99.2%	99.2%	99.2%	98.6%	98.1%	97.4%	95.4%
MinMDD	92.4%	92.1%	92.1%	90.8%	88.2%	86.9%	82.9%
MinCED	92.0%	92.0%	91.3%	89.7%	88.3%	86.4%	82.9%
RP_CED	97.9%	97.9%	96.7%	95.6%	92.7%	91.3%	89.4%
MinMCED	91.9%	91.9%	91.2%	90.3%	87.6%	85.1%	81.8%
RP_MCED	99.2%	99.2%	99.0%	98.3%	97.5%	96.3%	94.8%

Panel B. Mean

Portfolio Construction Approach	5%	10%	20%	30%	40%	50%	60%
RANDOM	0.095	0.104	0.124	0.147	0.177	0.205	0.218
EN	0.095	0.103	0.124	0.148	0.179	0.212	0.230
EVA	0.094	0.100	0.116	0.135	0.159	0.189	0.217
MinMDD	0.092	0.097	0.109	0.122	0.137	0.154	0.169
MinCED	0.093	0.097	0.109	0.123	0.139	0.157	0.173
RP_CED	0.094	0.102	0.119	0.140	0.166	0.193	0.210
MinMCED	0.092	0.097	0.109	0.122	0.137	0.156	0.175
RP_MCED	0.094	0.101	0.119	0.140	0.168	0.200	0.227

Panel C. Median

Portfolio Construction Approach	5%	10%	20%	30%	40%	50%	60%
RANDOM	0.095	0.103	0.124	0.147	0.173	0.200	0.209
EN	0.095	0.104	0.124	0.147	0.176	0.205	0.219
EVA	0.094	0.100	0.116	0.133	0.153	0.178	0.207
MinMDD	0.092	0.097	0.108	0.119	0.130	0.144	0.156
MinCED	0.093	0.097	0.108	0.121	0.135	0.147	0.158
RP_CED	0.094	0.101	0.119	0.138	0.159	0.182	0.189
MinMCED	0.092	0.096	0.106	0.117	0.129	0.141	0.155
RP_MCED	0.094	0.101	0.119	0.139	0.162	0.191	0.208

Figure 1. This figure displays distribution of deviations from optimal portfolios caused by sample error.

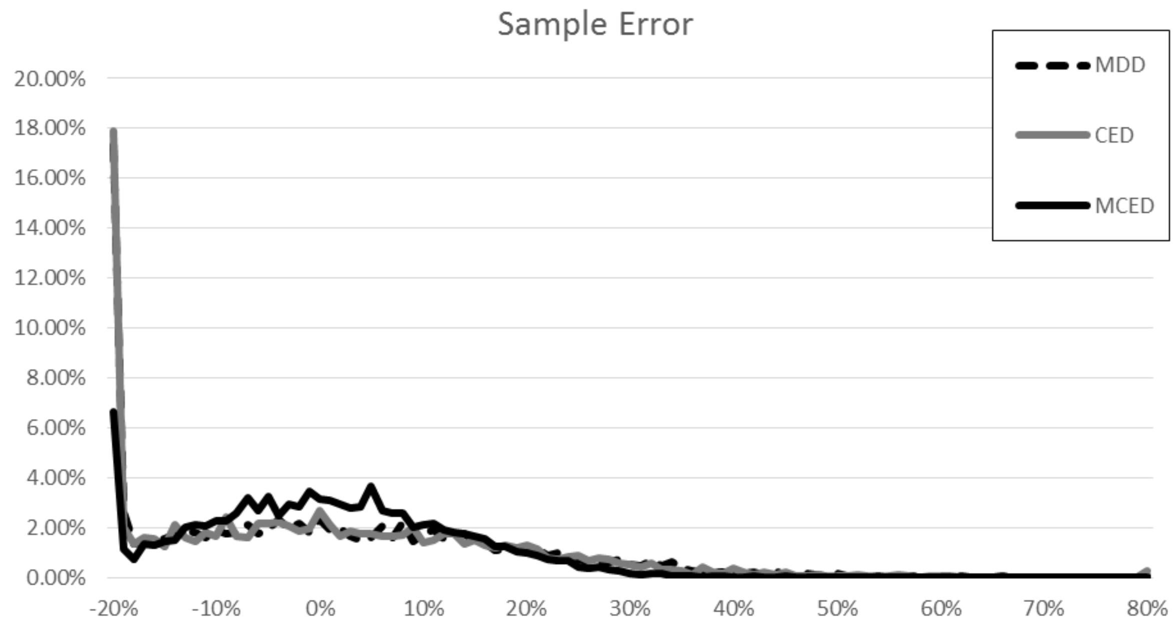

Figure 2. This figure shows performance of the 60--40 Portfolio of Stocks and Bonds for the period between January 1999 and June 2015. The portfolio is constructed using S&P 500 Total Return Index and JP Morgan Global Government Bond Index.

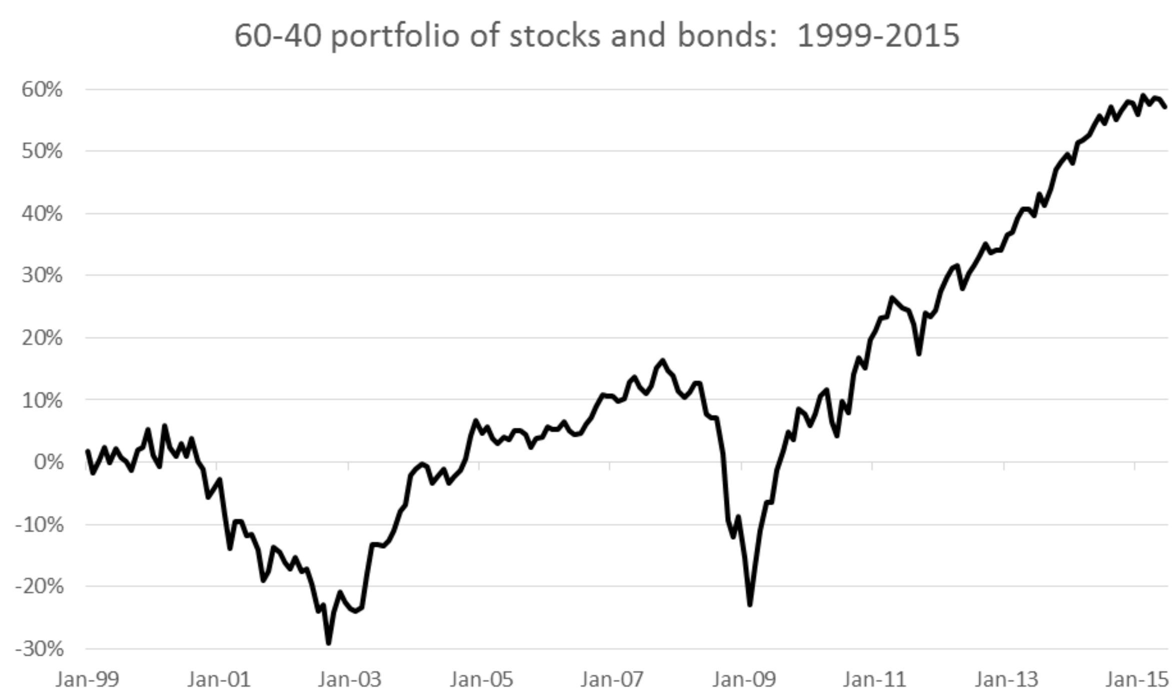

Figure 3. This figure displays distributions of the Sharpe ratios of hypothetical portfolios, generated within the large scale simulation framework for the out-of-sample period between January 1999 and June 2015.

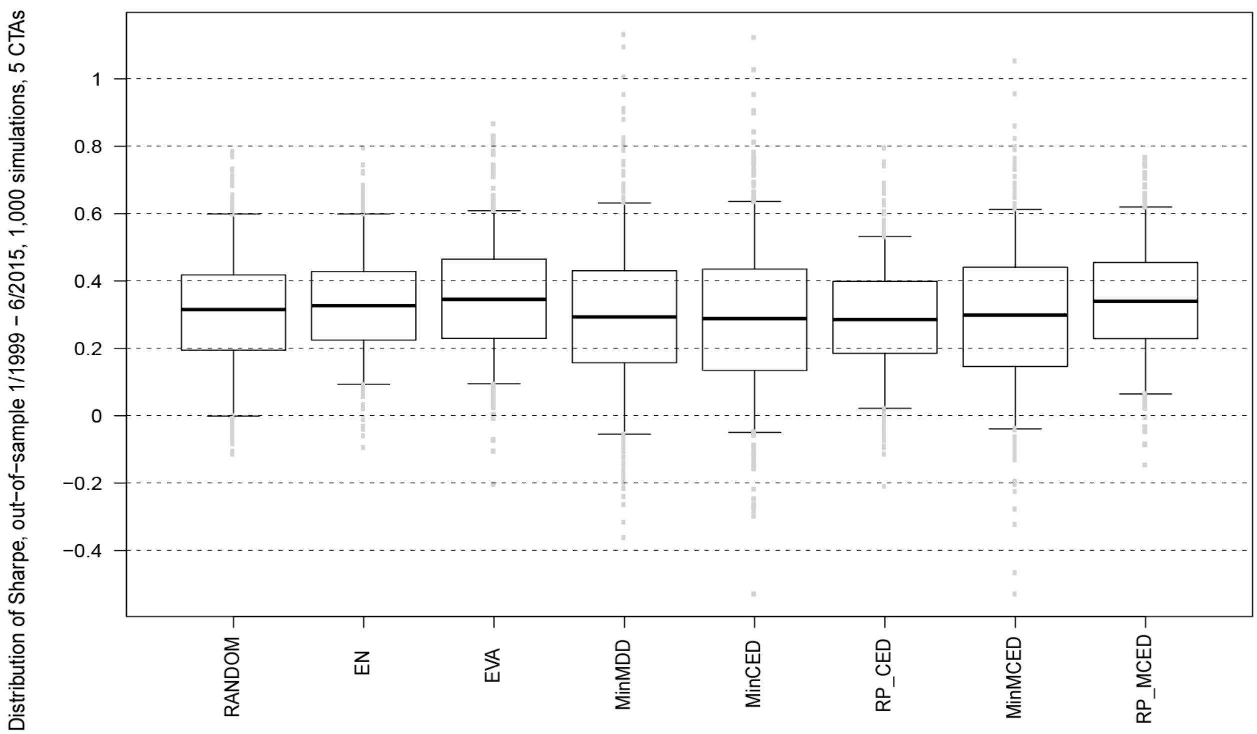

Figure 4. This figure displays distributions of the Calmar ratios of hypothetical portfolios, generated within the large scale simulation framework for the out-of-sample period between January 1999 and June 2015.

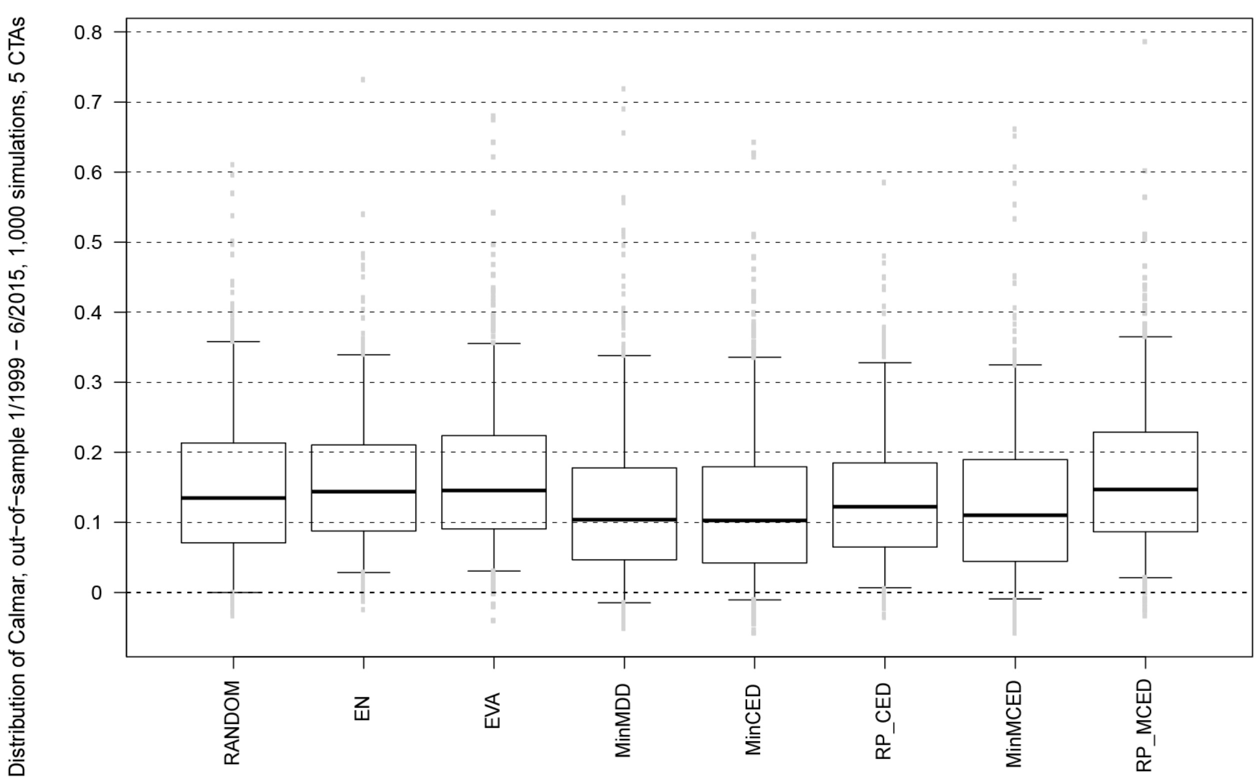

Figure 5. This figure shows distributions of the Sharpe ratios of blended portfolios of the original 60-40 portfolio and the hypothetical portfolios, generated using the MCED-based equal-risk allocation approach within the large scale simulation framework for the out-of-sample period between January 1999 and June 2015.

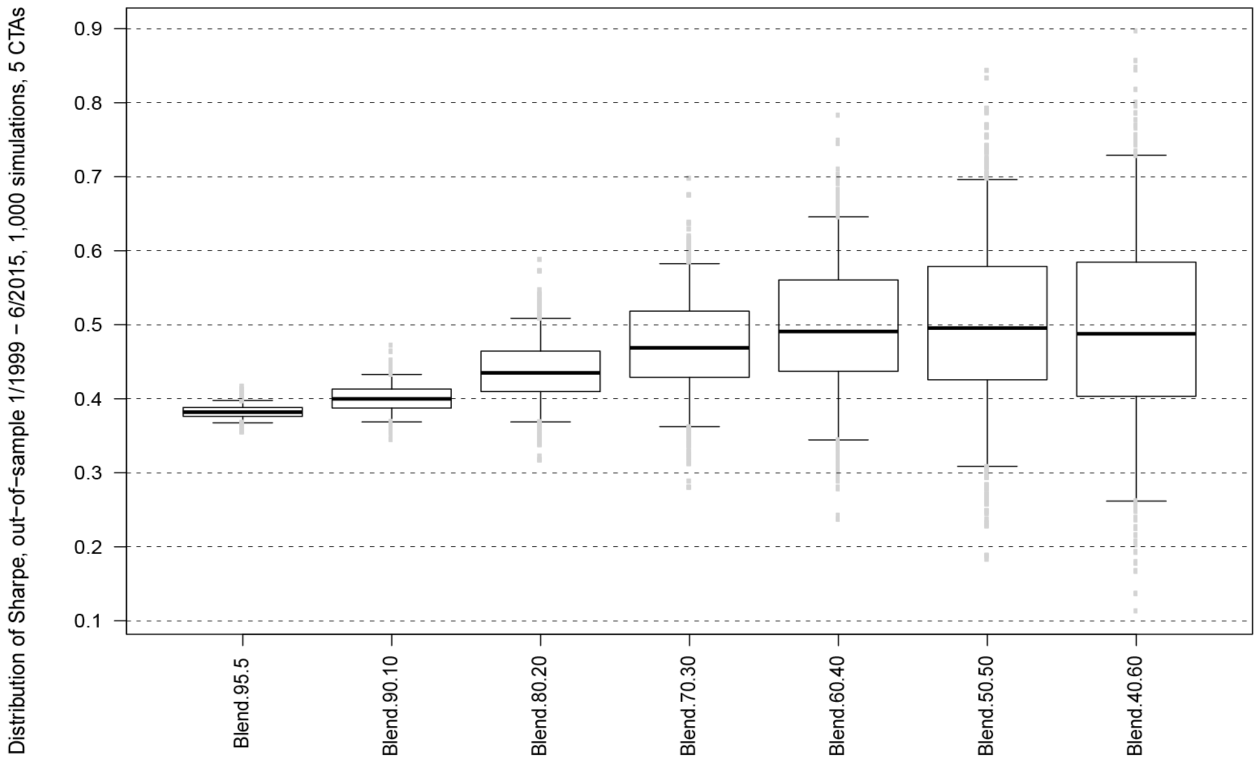

Figure 6. This figure shows distributions of the Calmar ratios of blended portfolios of the original 60-40 portfolio and the hypothetical portfolios, generated using the MCED-based equal-risk allocation approach within the large scale simulation framework for the out-of-sample period between January 1999 and June 2015.