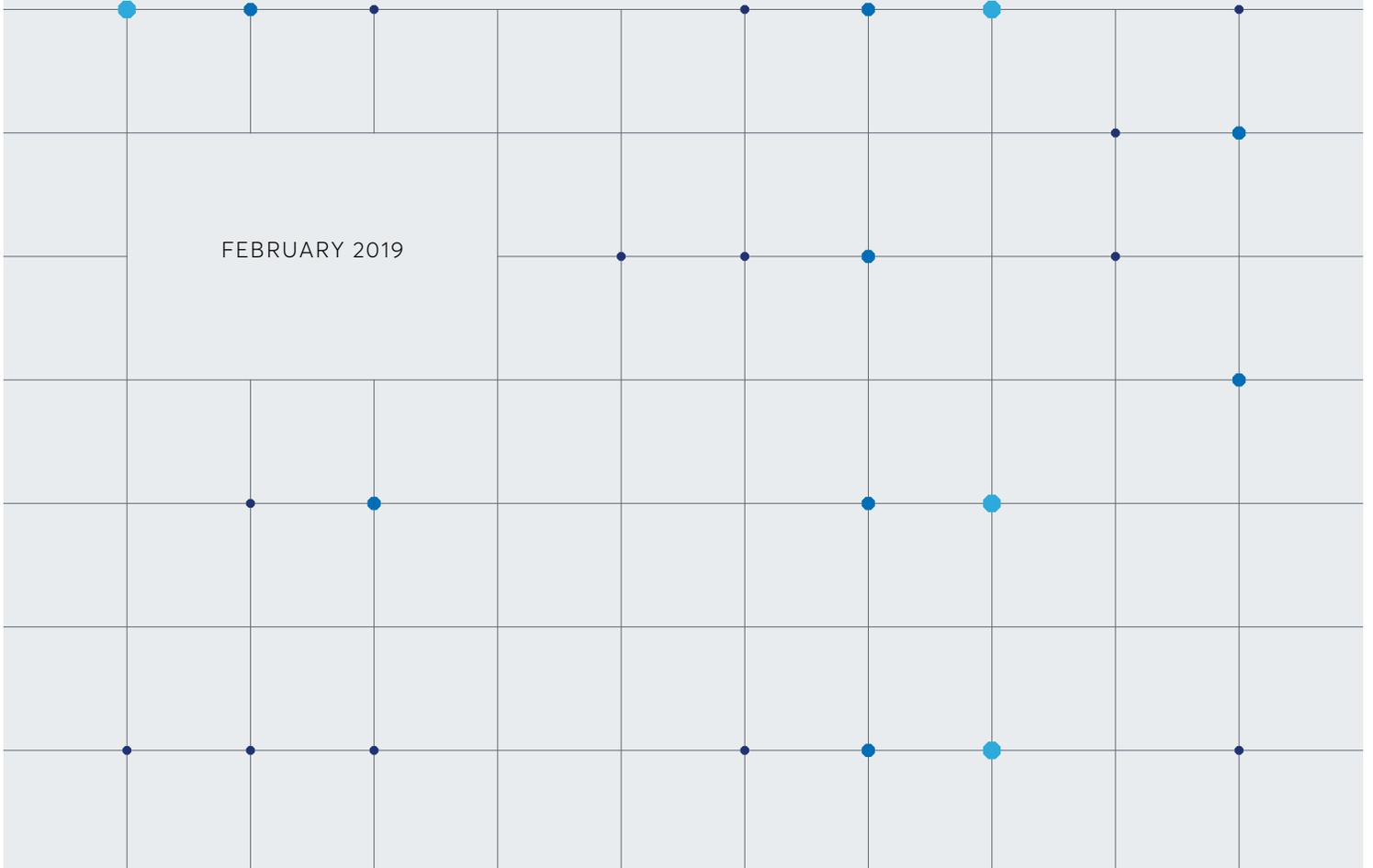


Eurodollar Futures: The Basics

INTEREST RATES



CME Eurodollar futures have reigned for decades as the most flexible, highly traded, and widely used of all listed interest rate derivatives. This user's guide spells out their basics: how they work, how they trade, how they relate to adjacent money markets.

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Three-Month Eurodollar Futures Contract Terms

Exhibit 1 summarizes key features of the Three-Month Eurodollar ("GE") futures contract. The following passages discuss, in order, the contract interest rate, the contract price mechanism, the schedule of contract delivery months and last trading days, minimum price increments for trading, and contract size.

Exhibit 1 – CME Three-Month Eurodollar Futures Contract Specifications

(All times of day are Chicago time unless otherwise noted.)¹²

Contract Unit	\$2,500 x contract IMM Index ¹												
Price Basis	Contract IMM Index = 100 minus contract interest rate Contract interest rate = interest rate per annum on three-month unsecured bank borrowing, for spot settlement on 3rd Wednesday of contract Delivery Month. (Eg, a contract price of 97.45 IMM Index points signifies a contract interest rate of 2.55 percent per annum.)												
Price Increments	<i>Nearest Delivery Month</i> ¼ of one interest rate basis point ("bp") = 0.0025 IMM Index points = \$6.25 per contract. (On Last Trading Day in an expiring nearest contract, the next nearest contract begins trading in 0.0025 increments.) <i>All other Delivery Months</i> ½ of one interest rate bp = 0.005 index points = \$12.50 per contract.												
Delivery Months	Generally, nearest 40 months in March Quarterly cycle. Generally, nearest 4 months not in March Quarterly cycle.												
Last Trading Day	2nd London bank business day before 3rd Wednesday of Delivery Month												
Delivery Standard	Three-month US dollar ICE LIBOR [®] set on Last Trading Day, for spot (T+2) settlement on 3rd Wednesday of Delivery Month, rounded to nearest 1/100th of one bp. ²												
Delivery Method	Cash settlement, by mark-to-market to final settlement price. <i>Final settlement price</i> = Price Basis evaluated at Delivery Standard. Final settlement occurs on Last Trading Day.												
Trading Hours	5pm to 4pm, Sun-Fri. Trading in expiring contracts terminates at 11am London time on Last Trading Day.												
Position Accountability	10,000+ contracts												
Reportable Positions	850+ contracts												
Block Trade Minimum Sizes	Nearest 20 March Quarterly Delivery Months and all Delivery Months not in March Quarterly cycle: <table style="float: right; margin-left: 20px;"> <tr> <td colspan="2">Farthest 20 March Quarterly Delivery Months:</td> </tr> <tr> <td>ATH (4pm to 12am)</td> <td>1,000 contracts</td> <td>250 contracts</td> </tr> <tr> <td>ETH (12am to 7am)</td> <td>2,000 contracts</td> <td>500 contracts</td> </tr> <tr> <td>RTH (7am to 4pm)</td> <td>4,000 contracts</td> <td>1,000 contracts</td> </tr> </table>		Farthest 20 March Quarterly Delivery Months:		ATH (4pm to 12am)	1,000 contracts	250 contracts	ETH (12am to 7am)	2,000 contracts	500 contracts	RTH (7am to 4pm)	4,000 contracts	1,000 contracts
Farthest 20 March Quarterly Delivery Months:													
ATH (4pm to 12am)	1,000 contracts	250 contracts											
ETH (12am to 7am)	2,000 contracts	500 contracts											
RTH (7am to 4pm)	4,000 contracts	1,000 contracts											
Designated Contract Market	CME Three-Month Eurodollar futures are listed for trading on, and are subject to the rules and regulations of, CME.												
Product Code	CME Globex: GE CME ClearPort: ED Clearing: ED Bloomberg: ED												

Source: CME Group

¹ When introduced in 1981, Three-Month Eurodollar futures were listed for trading under the auspices of what was then the International Money Market ("IMM") division of Chicago Mercantile Exchange. The "100 minus rate" contract pricing engine, now widely used by futures exchanges around the globe, came thus to be known as the IMM Index.

² ICE LIBOR[®] is a registered trademark of Intercontinental Exchange Holdings, Inc. and is used under license.

Contract Interest Rate

The price of any GE contract is an index number based on the contract interest rate.³ The contract interest rate, in turn, is the rate per annum:

- on a hypothetical *unsecured bank funding deposit*
- for *spot settlement* on the third Wednesday of the contract delivery month,
- with term to maturity of *three months*,
- for which *interest accrues* on an actual/360-day basis,
- and for which each basis point per annum of the corresponding rate of interest is worth \$25.

Though they seem obvious, the elements of this definition reward scrutiny.

Unsecured Bank Funding Deposit

The contract interest rate is assumed to apply to a US dollar-denominated wholesale funding deposit booked with a bank (or bank branch) domiciled in any of various funding centers, including Australia, Canada, the Cayman Islands, the European Free Trade Area, the European Union, Hong Kong, Japan, Singapore, and the United States.⁴

Spot Settlement

...means T+2. The settlement date (when borrower receives lender's monies) is assumed to occur two London bank business days after the trade date (when borrower and lender agree the deposit's amount and rate of interest). The settlement date must also be a New York bank business day. If it is not, then the deposit settles on the next following day on which banks are open for business in both London and New York. Three months later, the bank placement matures. As with settlement date, maturity date must be a bank business day in both London and New York.⁵

Example 1

A three-month bank placement is agreed on Monday, 18 February, for settlement on the third Wednesday of the month, 20 February. On 20 May, borrower repays lender principal plus interest, provided that 20 May is a London and New York bank business day. If it is not, then repayment is scheduled to mature on the next following London and New York bank business day.

³ See "Price = 100 Minus Contract Interest Rate", page 5.

⁴ The LIBOR benchmark administrator, ICE Benchmark Administration Ltd ("IBA"), maintains a list of approved funding centers. A bank that serves as a member of the USD LIBOR panel may use information that arises from funding transactions in these centers as data in formulating its daily submissions to IBA for USD LIBOR settings. The list of approved funding centers is reviewed periodically by IBA, in conjunction with the LIBOR contributor banks and the LIBOR Oversight Committee. For more information, see ICE Benchmark Administration, ICE LIBOR® Methodology, Section 3, Appendix B, "LIBOR input data", available at: https://www.theice.com/publicdocs/ICE_LIBOR_Methodology.pdf

⁵ As a general principle, if it is not a London-NY business day, the deposit's maturity date is moved to the next following London-NY business day, or advanced to the first preceding London-NY business day, depending on whether the maturity date falls near a month end. For details, see "Appendix: Day Count Conventions for Money Market Transactions" on page 34. As a practical matter, for the bank deposit corresponding to the interest rate that stands as underlying reference for a GE futures contract, the maturity date almost always is moved to the next following London-NY business day.

How Long is Three Months?

In regular years the interval from 20 February to 20 May spans 89 days. In leap years it covers 90 days, because it encompasses February's extra 29th day. By contrast, the three months from 20 May to 20 August always span 92 days.

Clearly, three-month intervals are not created equal. Given the market conventions that determine settlement and maturity dates for bank funding transactions, and given the peculiarities of bank holiday calendars in England and Wales and the US, "three months" may be as brief as 86 days or as long as 96 days. Even if the field is narrowed to three-month intervals that start on the third Wednesday of each month, day counts historically range from 89 to 92 days. 92 days is by far the most common outcome, occurring in 58 percent of cases (Exhibit 2).

The day count – the precise length of the bank funding deposit's three-month term – matters because it bears directly upon the interest amount that borrower is assumed to pay to depositor at maturity.

Exhibit 2 – Day Counts for US Dollar 3-Month Term Wholesale London Bank Deposits for Settlement on 3rd Wednesday of Month, Oct 1977 through Sep 2018

Length of 3-Month Term (Days)	Number of Occurrences	Pct Share of Occurrences
89	31	6
90	72	15
91	102	21
92	287	58

Interest Accrual

The interest rate per annum which lender and borrower agree is assumed to accrue on an actual/360 basis, with the actual number of days between settlement date and maturity date applied to a 360-day year.

Example 2

On Monday, 18 February, lender agrees to deposit \$1 million with borrower at a rate of 5 percent per year for three months, for standard spot settlement. Two London bank business days later, on Wednesday, 20 February, lender remits \$1 million to borrower.⁶ Assuming February has 28 days, borrower repays lender \$1 million principal plus interest 89 days later, on Monday, 20 May. The interest amount is:

$$\$12,361.11 = (\$1 \text{ million}) \times (5 \text{ percent per year}) \times (89 \text{ days} / 360 \text{ days per year})$$

Example 3

If the loan in Example 2 is undertaken in a leap year, then borrower repays lender principal plus interest 90 days later, on Tuesday, 20 May. The interest amount is:

$$\$12,500.00 = (\$1 \text{ million}) \times (5 \text{ percent per year}) \times (90 \text{ days} / 360 \text{ days per year})$$

⁶ Reminder: Wednesday, 20 February, also must be a bank business day in New York, as we assume here.

Price = 100 Minus Contract Interest Rate

GE futures prices are quoted in IMM Index (or "100 minus rate") terms. Price is expressed on the basis of 100 index points, with each index point representing one percent (ie, 100 basis points) per annum of contract interest rate exposure.

At Futures Expiration – Final Settlement Price

"Delivery" on expiring futures takes place not by physical delivery and acceptance of a three-month bank funding deposit, but rather by cash settlement. In effect, the contract is fulfilled by a final mark-to-market to its final settlement price.

The final settlement price is determined as 100 minus the three-month US dollar ICE LIBOR® value set on the second London bank business day preceding the third Wednesday of the contract delivery month.⁷ ICE LIBOR® is intended to provide an indication of the average rate at which LIBOR panel banks could obtain wholesale, unsecured funding for set maturities in particular currencies.⁸

The benchmark administrator, ICE Benchmark Administration Ltd ("IBA"), publishes each ICE LIBOR® value, including three-month US dollar ICE LIBOR®, to five decimal places of precision, ie, 1/1,000th of one bp per annum. The exchange then rounds the published value to four decimal places before using it to establish the final settlement price of the expiring GE futures contract.⁹ In the case of a tie, ie, a published value ending in 0.00005, the exchange rounds up to four decimal places. The exchange then subtracts this interest rate, so rounded, from 100 to determine the contract final settlement price in accord with the IMM Index standard.

Example 4

A published three-month US dollar ICE LIBOR® value of 3.14149 percent would be rounded by the exchange to 3.1415 percent before application to final settlement of an expiring GE future. A published value of 3.14145 percent also would get rounded up to 3.1415 percent. In either case, the resultant futures final settlement price would be 96.8585, equal to 100 minus 3.1415 percent.

Before Futures Expiration – Contract Price and Contract Interest Rate

For any GE futures contract, at any moment during the trading day, 100 minus the contract price is approximately the market consensus expectation of the contract interest rate.¹⁰

Example 5

If a GE contract is priced at 93.670 five months prior to expiration, the implication is that market participants collectively expect the interest rate on three-month wholesale bank funding placements to be approximately 6.330 percent per annum for standard spot settlement five months hence.

In this connection, two features of the IMM Index scheme are noteworthy. One is that it ensures a loose but intuitively appealing imitation of the inverse relationship between a bond's price and its yield. If a GE contract's reference interest rate rises, the contract price falls, and vice versa. A change of one interest rate basis point is always equal in magnitude, and opposite in sign, to a futures price change of 0.01 price points (Exhibit 3).

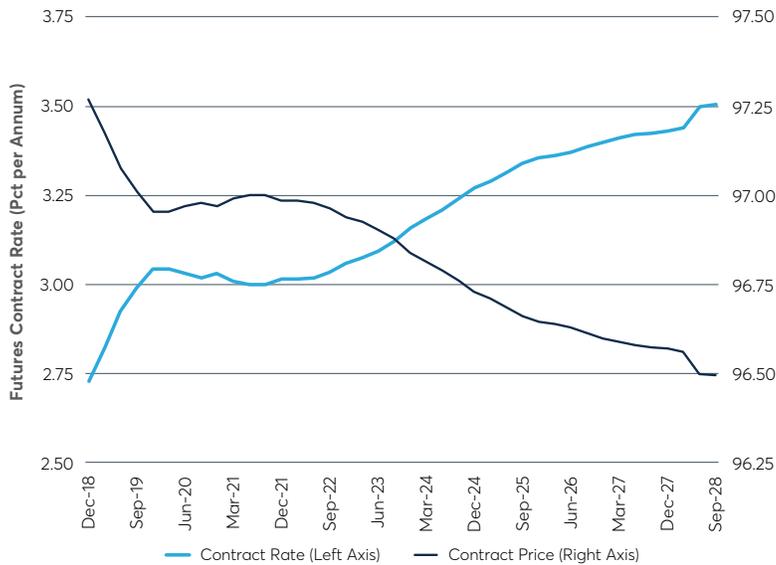
7 For information on the ICE LIBOR® methodology, see ICE Benchmark Administration, ICE LIBOR® Methodology, available at: https://www.theice.com/publicdocs/ICE_LIBOR_Methodology.pdf

8 See "ICE LIBOR® Output Statement", available at: https://www.theice.com/publicdocs/ICE_LIBOR_Output_Statement.pdf

9 This ensures that the final settlement price will produce a final mark-to-market for which the futures minimum price increment is worth 25 cents per contract. Without rounding, the final mark-to-market would be measured in unmanageably tiny increments of 2½ pennies per contract.

10 To see why it is "approximately" instead of exactly so, see "Almost a Forward Rate, but Not Quite: Convexity Bias", page 24.

Exhibit 3 – GE Futures Prices and Contract Interest Rates, Monday, 19 November 2018



Source: CME Group

The second is the flexibility of the Index mechanism. The futures price can exceed 100, if market participants broadly expect the contract interest rate to be set at negative levels (as has occurred in recent years in, eg, Eurozone, Japanese, and Swiss money markets).¹¹

Delivery Month And Last Trading Day

Each GE contract ceases trading and comes to final settlement on the second London bank business day (typically the Monday) before the third Wednesday of its delivery month. The exchange usually lists GE contracts for trading in each of 44 delivery months.

40 Quarterlies...

Quarterly contracts are listed for trading in each of the nearest 40 March Quarterly delivery months, ie, every March, June, September, and December in the coming decade.¹²

Example 6

GE futures for delivery in June 2019 are scheduled to cease trading and to expire on Monday, 17 June. When the CME Globex electronic trading platform ("CME Globex") session for trade date 17 June commences on Sunday afternoon, 16 June, trading is scheduled to open in the newly-listed GE futures for delivery ten years hence, in June 2029.

¹¹ The CME Globex electronic trading platform ("CME Globex") accommodates GE futures price levels that exceed 100 IMM Index points. It does not, however, permit price levels below zero. It is conceivable that IMM Index values could extend below zero, if market participants generally expected the contract interest rate to exceed 100 percent per year. In practice, this has occurred only rarely, in settings other than US money markets, eg, Russian bank loan markets in the early 1920s, or Chile in the mid-1970s. (See Sidney Homer, *A History of Interest Rates*, 2nd Ed, Rutgers University Press, 1977, Chapters 24 and 27.)

¹² Just before the nearby Quarterly contract expires, a new fortieth Quarterly contract is listed. Thus, for a few hours every March, June, September, or December – typically from the opening of the CME Globex trading session at 5pm Chicago time on the Sunday before the third Wednesday, until 11am London time on the ensuing Monday – 41 Quarterly GE futures are available for trading instead of 40.

...and 4 Monthly Serials

Monthly Serial GE contracts are identical to Quarterlies, except that they are for delivery months other than March, June, September, or December.¹³ The exchange generally lists Monthly Serial contracts for trading in each of the nearest four non-March Quarterly delivery months.¹⁴

Example 7

On Monday, 19 November 2018, Monthly Serial GE futures were listed for trading in each of the November 2018 and January, February, and April 2019 delivery months. Additionally, because that Monday was the last day of trading in the November 2018 contract, trading commenced in the new Monthly Serial contract for the May 2019 delivery month, when the CME Globex session for Monday opened at 5pm Chicago time (CT) on the previous Sunday afternoon, 18 November 2018.

Color Coding

For ease of identification the 40 Quarterly delivery months are grouped into ten quadruplets. Each quadruplet is identified by its own unique (and vivid) color code. In order of proximity to final settlement, these are White, Red, Green, Blue, Gold, Purple, Orange, Pink, Silver, and Copper.

On Monday, 19 November 2018, for instance, the White year included the December 2018 and March, June, and September 2019 delivery months, the Red year comprised the fifth through eighth nearest Quarterly delivery months – here, December 2019 and March, June, and September 2020 -- and so on (Exhibit 4).

¹³ A common point of confusion is the distinction between Monthly Serial GE futures and Monthly Serial options on Quarterly GE futures. Apart from the superficial similarity of expiring in months other than March, June, September, or December, they have no bearing upon each other. The futures contract into which any Monthly Serial GE option is exercisable is the GE future for delivery in the March Quarterly month next following the option's expiration month. For instance, October or November Monthly Serial GE options are exercisable only into GE futures for delivery the next following December, while January or February Monthly Serial GE options are exercisable only into GE futures for delivery the next following March. The Exchange lists no options that are exercisable into Monthly Serial GE futures.

¹⁴ Similar to Quarterly contracts, for a few hours on the last day of trading in any Monthly Serial GE contract, five non-March Quarterly delivery months are available for trading instead of four, because a new Monthly Serial contract is listed for trading at the start of that CME Globex trading session on the preceding evening.

Exhibit 4 – GE Futures Prices, Trading Volumes, and Open Interest, Monday, 19 Nov 2018
(Shaded rows denote Monthly Serial contract delivery months)

Delivery Month	Delivery Year	Color Code	Open Price	High Price	Low Price	Last Price	Settlement Price	Change vs Previous Settlement Price	Volume (contracts)	Open Interest (contracts)
NOV	2018		97.355	97.355	97.350	97.3525	97.3542	—	5,071	203,352
DEC	2018		97.2725	97.275	97.2575	97.270	97.270	0.005	403,006	1,620,917
JAN	2019		97.235	97.250	97.235	97.245	97.245	0.010	15,018	100,301
FEB	2019		97.205	97.225	97.205	97.220	97.215	0.015	3,298	32,829
MAR	2019		97.175	97.185	97.150	97.170	97.175	0.015	328,618	1,367,917
APR	2019		—	97.150 B	—	97.150 B	97.150	0.025	0	7,529
MAY	2019		97.115	97.115	97.115	97.115	97.100	0.015	14	0
JUN	2019		97.065	97.085	97.030	97.070	97.075	0.030	294,510	1,264,111
SEP	2019		96.990	97.020	96.955	97.005	97.010	0.040	284,790	1,174,319
DEC	2019	Red	96.930	96.965	96.895	96.950	96.955	0.040	541,164	2,084,570
MAR	2020		96.935	96.970	96.895	96.950	96.955	0.040	237,879	1,101,447
JUN	2020		96.945	96.980	96.910	96.965	96.970	0.040	252,190	1,022,846
SEP	2020		96.955	96.985	96.920	96.975	96.980	0.040	240,883	782,403
DEC	2020	Green	96.950	96.980	96.910	96.965	96.970	0.035	231,832	1,214,134
MAR	2021		96.970	97.000	96.930	96.985	96.990	0.035	101,715	582,465
JUN	2021		96.985	97.010	96.945	96.995	97.000	0.035	88,620	431,499
SEP	2021		96.985	97.010	96.950	96.995	97.000	0.030	114,022	267,743
DEC	2021	Blue	96.975	96.995	96.935	96.980	96.985	0.025	92,116	390,823
MAR	2022		96.970	96.990	96.940	96.980	96.985	0.025	44,276	165,143
JUN	2022		96.965	96.985	96.935	96.970	96.980	0.025	43,846	150,510
SEP	2022		96.960	96.970	96.925	96.960	96.965	0.020	40,269	105,185
DEC	2022	Gold	96.935	96.950	96.905	96.935	96.940	0.015	40,505	119,720
MAR	2023		96.920	96.935	96.890	96.920	96.925	0.015	19,033	67,001
JUN	2023		96.905	96.915	96.870	96.900	96.905	0.015	16,562	52,366
SEP	2023		96.875	96.890	96.845	96.875	96.880	0.015	18,269	48,231
DEC	2023	Purple	96.850	96.850	96.810	96.840 A	96.840	0.010	211	13,040
MAR	2024		96.795	96.825	96.790	96.815 A	96.815	0.005	219	8,393
JUN	2024		96.770	96.795	96.765	96.790 A	96.790	0.005	399	4,695
SEP	2024		96.740	96.765	96.735	96.765	96.760	0.005	172	3,224
DEC	2024	Orange	96.715	96.720	96.710	96.720	96.730	0.005	11	1,854
MAR	2025						96.710	0.005	0	512
JUN	2025						96.685	unch	12	1,124
SEP	2025						96.660	unch	26	460
DEC	2025	Pink					96.645	unch		520
MAR	2026						96.640	unch		363
JUN	2026						96.630	unch		233
SEP	2026						96.615	unch		41
DEC	2026	Silver					96.600	unch		78
MAR	2027						96.590	unch		78
JUN	2027						96.580	unch		11
SEP	2027						96.575	unch		108
DEC	2027	Copper					96.570	unch		21
MAR	2028						96.560	unch		
JUN	2028						96.500	unch		
SEP	2028						96.495	unch		
Totals									3,458,556	14,392,116

B ~ Bid Price. A ~ Offered Price. Source: CME Group.

Nearby vs Lead or Front

The "nearby" is simply the GE futures contract that expires next. It can be either a Quarterly or a Monthly Serial. Context matters, however. Market practitioners sometimes use "nearby" as a short-hand reference for the next expiring Quarterly.

"Lead" or "front" refers to the most actively traded Quarterly contract. In the heyday of open outcry trading, the "lead" designation served to aid the exchange's Eurodollar futures pit committee and trading floor operations staff in deciding which contract delivery month would take pride of place within the then massive Eurodollar futures pit. Although nearly all volume is now mediated through CME Globex, the "lead month" label lives on as a convenient nickname for the most actively traded delivery month.

Contract Size And Minimum Price Increment

One Basis Point = \$25

GE futures product rules implement the IMM Index so that each 1/100th of one Index point ("tick") is worth \$25 per contract or, equivalently, each IMM Index point is worth \$2,500 per contract. It follows that the dollar value of each basis point per annum ("DV01") of the contract interest rate is likewise worth \$25 per contract. Crucially, this applies regardless of the length of time until the contract expires.¹⁵

A straightforward approach to characterizing the size of a GE futures contract (or, for that matter, the size of any stock price or commodity price index futures product) is based on the notion of "contract equity": (contract price) x (contractual value per price point). For a GE futures contract, contract equity = (contract IMM Index) x (\$2,500 per contract IMM Index point).

Example 8

The contract equity in a position in a GE futures contract priced at 93.670 points is \$234,175 per contract, equal to (93.670 price points) x (\$2,500 per price point per contract).

Minimum Price Increment = ½ Tick or ¼ Tick

With one exception, the minimum price movement for trading GE futures is one half of one interest rate basis point (0.005 contract price points or ½ tick), equal to \$12.50 per contract.

The exception is the nearby contract for which the minimum price movement is one quarter of one interest rate basis point (0.0025 contract price points or ¼ tick), equal to \$6.25 per contract. On the eve of the last day of trading in an incumbent nearby contract (either Quarterly or Monthly Serial), the contract for the next following delivery month (whether Quarterly or Monthly Serial) begins trading in ¼ tick minimum price increments.

Example 9

The December 2018 Quarterly contract (GEZ18) becomes the nearby on Monday, 19 November 2018, the last day of trading in the November 2018 Monthly Serial contract (GEX18). The minimum price increment for GEZ18 graduates from ½ tick to ¼ tick at 5pm Chicago time on Sunday, 18 November, when the CME Globex session commences for the Monday, 19 November, trade date. Thus, the incumbent nearby, GEX18, and the successor nearby, GEZ18, simultaneously trade in ¼ tick minimum price increments for a few hours, until trading in the incumbent terminates at 11am London time (generally, 5am Chicago time). GEZ18 continues to trade in ¼ tick price increments until it too ceases trading and comes to final settlement on Monday, 17 December 2018, at which time nearby status passes to the January 2019 Monthly Serial (GEF19).

¹⁵ Why this is "crucial" is discussed in "Almost a Forward Rate, but Not Quite: Convexity Bias", page 24-32.

The Parable of the \$1 Million

When GE futures debuted in December 1981, two contract design features were completely unfamiliar:

- Futures contracts priced by reference to index numbers were unknown.¹⁶
- In an age when all futures contracts were fulfilled by physical delivery, it required a leap of faith for market participants to comprehend, let alone to use, futures that expire by cash settlement (ie, by final mark-to-market to an externally-produced price value).

To make GE futures comprehensible to bank treasurers and other prospective users, both the exchange and the brokerage community relied upon a heuristic description of the contract mechanism that linked it to an imaginary physically-deliverable bank funding deposit. For example: "The contract unit is \$1,000,000 3-month Eurodollar time deposits... The basic tick size is 0.01 (or 1 basis point...). The dollar value of a tick is \$25, which accords with the change in the value of a 90-day \$1,000,000 instrument."¹⁷ This heuristic rule of thumb can be formalized as:

$$\text{\$25 per bp} = (\text{\$1 million}) \times (0.01 \text{ pct per yr}) \times (90 \text{ days} / 360 \text{ days per yr})$$

As with any rule of thumb, if it is taken too literally, it obscures instead of illuminating. Indeed, it often poses a dilemma for GE futures users: Because contract terms fix the value of each price tick at \$25 per contract, it is frequently impossible for \$1 million to be the size of the hypothetical bank placement at the heart of the rule of thumb.

To see why, rearrange the equivalence above to highlight the relationship between the size of the hypothetical bank funding deposit and the day count of its three-month term:

$$\text{\$1 million} = (\text{\$25 per bp}) / \{ (0.01 \text{ percent per year}) \times (90 \text{ days} / 360 \text{ days per year}) \}$$

This re-expression confirms that if the three-month term spans exactly 90 days, then the principal amount of the bank placement is indeed \$1 million. Given the circumstances that matter for final settlements of GE futures contracts, however, a 90-day span makes a poor rule of thumb: It's uncommon, occurring in just 15 months out of every 100 (Exhibit 2).

Suppose instead that the three-month interval spans 92 days (which occurs far more frequently). For the DV01 of the imaginary bank funding transaction to remain fixed at \$25, as GE product rules require, the principal amount of the transaction would have to be reduced to around \$978,261:

$$\text{\$978,260.87} = (\text{\$25 per bp}) / \{ (0.01 \text{ percent per year}) \times (92 \text{ days} / 360 \text{ days per year}) \}$$

Alternatively, if the rule of thumb were built upon a three-month interval spanning 89 days, the principal amount of the imaginary bank placement would have to increase to around \$1,011,236 (Exhibit 5).

Exhibit 5 – Bank Funding Deposit Sizes and Day Counts

Day Count of 3-Month Bank Deposit	Deposit Size Required for DV01 to Equal \$25 (Dollars)
89	1,011,235.96
90	1,000,000.00
91	989,010.99
92	978,260.87

Source: CME Group

¹⁶ Consider that the first equity price index futures product, the Kansas City Board of Trade Value Line Index futures contract, was not introduced until March 1982, three months after the launch of CME GE futures.

¹⁷ Burghardt, Galen, *The Eurodollar Futures and Options Handbook*, McGraw Hill, 2003, pg 30.

In all three cases, the point remains the same:

- To say that a GE futures contract represents a bank funding transaction with principal value of approximately \$1 million is appealingly simple and arithmetically convenient. But it is nothing more.
- The size of a GE futures contract is determined solely by the requirement that each basis point per annum of contract interest is worth \$25, at all times, in all places.

Trading Eurodollar Futures

Of equal importance to the GE contract mechanism is how GE futures trade. The following passages survey:

- GE Pack and Bundle combination trades;
- other standardized GE spreads and combinations;
- how trade prices for spreads and combinations become GE contract prices;
- how CME Globex matches buyers and sellers; and
- the process by which the exchange establishes contract daily settlement prices.

Bundles And Packs

Many hedging and trading strategies call for purchase or sale of multiple GE contracts for consecutive delivery months ("strips"). Besides being cumbersome, executing each of a strip's component contracts is apt to incur slippage costs that hurt trade profitability. GE Bundles and Packs, standardized for trading on CME Globex, provide an efficient alternative.

Packs

A Pack is the simultaneous purchase or sale, in a single trade, of one each of four GE contracts with consecutive Quarterly delivery months. In theory, a Pack may be structured so that the nearest of its member contracts is any of the nearest 37 Quarterly delivery months. In practice, most traffic occurs in Packs that correspond to the ten color-coded delivery years, White through Copper, shown in Exhibit 4. For instance, a Blue Pack transaction on 19 November 2018 would entail one each of GE contracts for the 9th through 12th nearest Quarterly delivery months, December 2021 and March, June, and September 2022.

Bundles

A Bundle is the simultaneous purchase or sale of one each of eight or more GE contracts with consecutive Quarterly delivery months. Bundles come in yearly tenors, from two to 10 years. (There are no one-year Bundles, because Packs play this role.) As with Packs, the nearest member contract in a Bundle may be any Quarterly delivery month, as long as there is a sufficient number of more distant Quarterly delivery months to accommodate the Bundle's tenor. The most highly traded Bundles, however, tend to be those for which the nearest contract in the Bundle sequence is the nearby Quarterly (Exhibit 6).¹⁸

¹⁸ *Reminder: Exhibit 6 includes no nearby one-year Bundle, because the White-Year Pack already does this job.*

Exhibit 6 – Popular GE Futures Bundles

Bundle Tenor (Years)	Bundle = 1 Each of the Following Quarterly Contracts:	DV01 (Dollars)	DV per 1/4 Tick (Dollars)
2	Nearest 8	200	50
3	Nearest 12	300	75
4	Nearest 16	400	100
5	Nearest 20	500	125
6	Nearest 24	600	150
7	Nearest 28	700	175
8	Nearest 32	800	200
9	Nearest 36	900	225
10	Nearest 40	1,000	250
5-Year Forward	Back 20 (Purple to Copper Years)	500	125

Source: CME Group

Bundle and Pack Prices

The price of any Bundle or Pack trade is quoted as the arithmetic average of changes in prices of its component GE contracts versus their respective previous daily settlement prices. Such price changes are quoted in minimum increments of $\frac{1}{4}$ of one GE price tick ($\frac{1}{4}$ of one bp per annum of Pack or Bundle interest rate exposure).

For any Pack, the DV01 is \$100, and the dollar value of a $\frac{1}{4}$ tick price change is \$25. For Bundles, DV01s range from \$200 per basis point (\$50 per $\frac{1}{4}$ tick) for the Two-Year Bundle to \$1,000 per basis point (\$250 per $\frac{1}{4}$ tick) for the 10-Year Bundle.

How Bundle and Pack Prices Become GE Contract Prices

Once a Bundle or Pack is transacted, the price change at which it has traded is assigned to each of its member contracts, in increments of one price tick.¹⁹ If the Bundle or Pack trade price is an integer number of ticks (eg, -7 ticks on the day), then each constituent contract gets booked at a price equal to its previous daily settlement price adjusted by the Bundle or Pack price (eg, previous daily settlement price minus 7 ticks).

If the Bundle or Pack price involves a fraction ($\frac{1}{4}$ or $\frac{1}{2}$ or $\frac{3}{4}$) of a tick, then the exchange assigns prices to the component contracts as follows:

- (1) Each component is assigned a price change equal to the integer portion of the Bundle or Pack price.
- (2) Individual contract price changes are then adjusted, one by one. If the Bundle or Pack price is positive, the adjustment pushes the individual contract price change up by one tick. If the Bundle or Pack price is negative, the adjustment pushes the individual contract price change down by one tick.
- (3) Importantly, the adjustment process described in step (2) starts with the most distant delivery month among the component contracts, then works forward until the average price change among all component contracts matches the Bundle or Pack price

With the prices of its constituent contracts thus determined, the Bundle or Pack is promptly unbundled (or unpacked, as the case may be). From then on, each such contract is marked to market and otherwise treated as if it were a stand-alone GE futures position.

¹⁹ To learn how CME Globex matches buyer and seller to make the Bundle or Pack transaction in the first place, see "CME Globex Trade Matching Algorithms for GE Futures" on pages 20-23.

Example 10

The Two-Year Bundle trades at +2.25 ticks. To accommodate the ¼ tick portion of the price, the Bundle's six nearest contracts get booked at prices equal to their previous daily settlements plus 2 ticks, while the two most deferred contracts get booked at prices equal to their previous daily settlements plus 3 ticks. The average price increment among the Bundle's eight component contracts is as it should be:

$$+2.25 \text{ ticks} = \{ (6 \text{ nearest contracts} \times +2 \text{ ticks}) + (2 \text{ most deferred contracts} \times +3 \text{ ticks}) \} / 8 \text{ contracts}$$

The Ten-Year Bundle trades at -5.75 ticks. Among its 40 member contracts, each of the nearest 10 delivery months is assigned a net price change of -5 ticks, while each of the 30 most distant delivery months gets booked at a net price change of -6 ticks versus its previous daily settlement price.

$$-5.75 \text{ ticks} = \{ (10 \text{ nearest contracts} \times -5 \text{ ticks}) + (30 \text{ most deferred contracts} \times -6 \text{ ticks}) \} / 40 \text{ contracts}$$

The Purple Pack trades at +0.25 ticks. The contract with the most distant delivery month in the Pack gets booked at a net price change of +1 tick versus its previous daily settlement price. The nearest three delivery months get booked at prices unchanged from their respective previous daily settlements –

$$+0.25 \text{ ticks} = \{ (3 \text{ nearest contracts} @ \text{ unchanged}) + (\text{most deferred contract} \times +1 \text{ tick}) \} / 4 \text{ contracts}$$

Calendars, Flies, Condors, And Other Spreads

In addition to Packs and Bundles, eight other popular spread strategies are standardized on CME Globex, permitting market participants to transact them directly instead of having to leg into or out of them. These include calendar spreads, butterflies, double butterflies, condors, month-Pack spreads, Pack spreads, Pack butterflies, and Bundle spreads. In the following discussion, Examples 11 through 19 refer to market conditions on Monday, 19 November 2018, as shown in Exhibit 4.

For calendar spreads, butterflies, double butterflies, and condors, the building blocks are individual GE contracts.

Calendar Spread

...consists of two GE contracts for different delivery months. The ratio for purchase of a calendar spread is always +1:-1:

- (a) buy 1 nearer delivery month (Leg1) and
- (b) sell 1 more distant delivery month (Leg2).

The spread price is quoted as (Leg1 price) minus (Leg2 price). For any calendar spread in which at least one leg is either a Monthly Serial and/or the nearest Quarterly delivery month, the minimum price increment is one quarter of one spread basis point (0.0025 contract price points or ¼ tick), equal to \$6.25 per spread. For any other calendar spread the minimum price movement is one half of one calendar spread basis point (0.005 price points or ½ tick), equal to \$12.50 per spread.

Example 11

Consider a Jan19-Mar19 calendar spread. At opening price levels, with the Jan19 contract at 97.235 and Mar19 at 97.175, the spread would be fairly valued at +6 ticks (equal to 97.235 minus 97.175). At daily settlement price levels, the Jun20-Jun22 two-year calendar spread would be fairly valued at -1 tick (equal to 96.970 minus 96.980).

Butterfly

... comprises three Quarterly GE contracts with equally distributed delivery months. CME Globex enables trading in butterflies in each of four standardized configurations: three-month (eg, Jun20-Sep20-Dec20), six-month (eg, Jun20-Dec20-Jun21), nine-month (eg, Jun20-Mar21-Dec21), and one-year (eg, Jun20-Jun21-Jun22). In all instances, the ratio for purchase of a butterfly is +1:-2:+1:

- (a) buy 1 nearest delivery month (Leg1),
- (b) sell 2 second-nearest delivery month (Leg2), and
- (c) buy 1 most distant delivery month (Leg3).

Price is quoted as (Leg1 price) minus (2 x Leg2 price) plus (Leg3 price). Minimum price increment is always one half of one spread basis point (0.005 price points or ½ tick), equal to \$12.50 per spread.

Double Butterfly

... combines four Quarterly GE contracts with equally distributed delivery months. It's useful to imagine a double butterfly as a calendar spread between two butterflies, in which the two more distant legs of the nearer butterfly serve double duty as the two nearer legs of the more distant butterfly. Thus, for purchase of a double butterfly the spread ratio is always +1:-3:+3:-1:

- (a) buy 1 nearest delivery month (Leg1),
- (b) sell 3 second-nearest delivery month (Leg2),
- (c) buy 3 third-nearest delivery month (Leg3), and
- (d) sell 1 most distant delivery month (Leg4).

CME Globex customarily enables trading in three variants: three-month (eg, Jun20-Sep20-Dec20-Mar21), six-month (eg, Jun20-Dec20-Jun21-Dec21), and one-year (eg, Jun20-Jun21-Jun22-Jun23). In all cases, price is quoted as (Leg 1 price) minus (3 x Leg 2 price) plus (3 x Leg 3 price) minus (Leg 4 price). Minimum price movement is one half of one spread basis point (0.005 price points or ½ tick), equal to \$12.50 per spread.

Example 12

At daily settlement price levels, the Jun20 six-month double butterfly would be fairly valued at +7.5 ticks, equal to 96.970 minus (3 x 96.970) plus (3 x 97.000) minus 96.985.

Condor

... involves four Quarterly GE contracts with equally distributed delivery months. As with double butterflies, three species of condor are available for trading on CME Globex, distinguished from one another by breadth of wing span: three-month (eg, Jun20-Sep20-Dec20-Mar21), six-month (eg, Jun20-Dec20-Jun21-Dec21), and one-year (eg, Jun20-Jun21-Jun22-Jun23). The ratio for buying a condor is always +1:-1:-1:+1:

- (a) buy 1 nearest delivery month (Leg1),
- (b) sell 1 second-nearest delivery month (Leg2),
- (c) sell 1 third-nearest delivery month (Leg3), and
- (d) buy 1 most distant delivery month (Leg4).

Price is quoted as (Leg1 price) minus (Leg2 price) minus (Leg3 price) plus (Leg4 price). Minimum price movement is one half of one spread basis point (0.005 price points or ½ tick), equal to \$12.50 per spread.

Example 13

At daily settlement price levels, fair value in the Jun20 six-month condor would be -1.5 ticks, equal to 96.970 minus 96.970 minus 97.000 plus 96.985.

For the remaining four standardized combinations – month-Pack spreads, Pack spreads, Pack butterflies, and Bundle spreads – the elementary building blocks are largely Packs and Bundles, rather than individual GE contracts.

In each case, price is quoted in the same terms as for the component Packs or Bundles: The spread price is quoted as the net change on the day (versus previous daily settlement prices in GE futures) among the legs of the spread, and the minimum price movement is one quarter of one spread basis point (0.0025 price points or ¼ tick).

Uniquely for the month-Pack spread, this price expression format requires the price of the GE contract on the spread's singleton leg to be quoted not in the usual "100 minus rate" terms, but in terms of the "net-change-on-the-day" convention that applies to the Pack on the spread's other leg –

Month-Pack Spread

...consists of a position in a single Quarterly GE contract combined with a countervailing position in a Pack. The four GE contracts in the Pack leg are always the next four Quarterly delivery months immediately following the delivery month for the GE contract on the singleton leg. The spread ratio is (+4 singletons):(-1 pack). Thus, purchase of a month-Pack spread entails:

- (a) buying 4 of the (nearer) singleton GE delivery month and
- (b) selling 1 (more distant) Pack.

Example 14

At daily settlement price levels, the Sep22 month-Pack spread would be fairly valued at +0.5 ticks, equal to:

(net change of +2 ticks in the Sep22 GE contract – the fourth contract in the Blue delivery year – versus its previous daily settlement price)

minus

(+1.5 ticks in the Gold Pack).

Pack Spread

...is similar to the futures calendar spread described earlier, except built with Packs instead of individual futures contracts. Each of the spread's two Packs corresponds to one of the color-coded delivery years exemplified in Exhibit 4. This means that futures delivery months represented in the Pack on one leg are always removed from futures delivery months in the Pack on the other leg by an integer multiple of one year, ranging from one (eg, White-Red, Silver-Copper) to nine (uniquely, White-Copper). The spread ratio for purchase of a Pack spread is always +1:-1:

- (a) buy 1 Pack of GE contracts with nearer delivery months (Leg 1) and
- (b) sell 1 Pack of GE contracts with more distant delivery months (Leg 2).

Pack Butterfly

...is just a butterfly in which each leg is a Pack. Here too, each of the Packs in the butterfly must correspond to one of the ten color-coded GE futures delivery years. CME Globex permits trading in two species of Pack butterfly: one-year (eg, Green-Blue-Gold) and two-year (eg, Green-Gold-Orange). As with the futures butterfly, the spread ratio for purchase of a Pack butterfly is always +1:-2:+1. For instance, buying a one-year Pack butterfly means:

- (a) buy 1 Pack of GE contracts with four nearest delivery months (Leg 1),
- (b) sell 2 Packs of GE contracts with the next four nearest delivery months (Leg 2),
- (c) buy 1 Pack of GE contracts with the most distant four delivery months (Leg 3).

Example 15

Assume that, just prior to 2pm CT daily settlements, Red, Green, and Blue Packs are priced at +4.0 ticks, +3.5 ticks, and +2.5 ticks, respectively, in their characteristic "net-change-on-the-day" terms. The Red-Blue 2-year Pack spread would be fairly valued at a net change of +1.5 ticks, equal to (+4.0 ticks) minus (+2.5 ticks). The Red-Green-Blue one-year Pack butterfly would be fairly priced at a net change of -0.5 ticks, equal to (+4.0 ticks) minus (2 x +3.5 ticks) plus (+2.5 ticks).

Bundle Spread

...is a calendar spread between two Bundles in which:

- the Bundle on one leg must comprise at least two GE contract delivery months that are not represented in the Bundle on the other leg, and
- the same number of constituent GE contracts on each leg must be equal to the other. For instance, a Two-Year Bundle can be paired only with another Two-Year Bundle to make a Two-Year Bundle spread.

The ratio for purchase of a Bundle spread is always +1:-1:

- (a) buy 1 Bundle of GE contracts for nearer delivery (Leg 1) and
- (b) sell 1 Bundle of GE contracts for more distant delivery (Leg 2).

How Spread Prices Become GE Contract Prices

As with Pack and Bundle combinations, all spread strategies described here are ephemeral, in the sense that each loses its identity at the moment CME Globex assigns prices to its constituent GE contracts. From then on, each of the GE contracts comprised within the spread is marked-to-market and otherwise treated as if it were a distinct GE futures position. Before we describe the process of price assignment, an item of terminology requires introduction:

C-Last Price

The CME Last ("C-Last") price for a GE contract for a given delivery month is the contract's latest trade price, or actionable price indication, or settlement price. Specifically, it is the most recent of:

- (a) latest CME Globex transaction price, or
- (b) CME Globex bid price that betters the bid side of the market, or
- (c) CME Globex asking price that betters the ask side of the market, or
- (d) latest daily [settlement price](#).

For this purpose, a bid that betters the market is understood to be a bid to buy at a higher price than the incumbent C-Last price. Similarly, a better ask price is understood to be an offer to sell at a price below the prevailing C-Last price.

The eight spread strategies fall into three groups distinguished by how CME Globex translates their trade prices into prices for their constituent GE contracts:

- calendar spreads, butterflies, double butterflies, and condors, on one hand; and
- Pack calendar spreads, Pack butterflies, and Bundle spreads on the other hand; with
- month-Pack spreads as a hybrid case.

For a calendar spread, butterfly, double butterfly, or condor, price is distributed to the component GE contracts on the basis of current market conditions in those contracts, as reflected in their C-Last prices.

Calendar Spread = (+1 Leg1):(-1 Leg2)

CME Globex checks which leg has the fresher C-Last price during the current trading session, then assigns that C-Last price to that leg. This result is combined with the calendar spread's transaction price to derive the price of the other leg.

If Leg1's C-Last price is fresher than Leg2's, then
 Leg1 price = its C-Last price, and
 Leg2 price = (Leg 1 price) minus (calendar spread price).

If Leg2 has the fresher C-Last price, then
 Leg2 price = its C-Last price, and
 Leg1 price = (Leg 2 price) plus (calendar spread price).

If the two C-Last prices happen to have been set at the same instant, Leg1 takes precedence: It is assigned its C-Last price, and Leg 2's price is derived as (Leg 1 price) minus (calendar spread price).

If there have been no trades in the Leg1 or Leg2 contracts during the current CME Globex trading session, then Leg1 again takes pride of place:

Leg1 price = Leg1's latest daily settlement price, and
 Leg2 price = (Leg1 price) minus (calendar spread price).

Examples 16

Suppose the Jun20-Jun22 two-year calendar spread trades at +1 ticks. Suppose that, of the two legs, the fresher C-Last price is in the Jun22 (GEM22) contract, at 96.97. For the spread buyer, CME Globex confirms the sale of one GEM22 at its C-Last price of 96.97 and the purchase of one Jun20 (GEM20) contract at an imputed price of 96.98, equal to GEM22 price plus the spread price of +1 tick. Conversely, the spread seller receives confirmation of sale of one GEM20 at 96.98 and purchase of one GEM22 at 96.97.

What if determination of the GEM22 C-Last price, 96.97, happens to coincide with determination of the GEM20 C-Last price, 96.965? Then the contract for the nearer delivery month takes precedence: The spread buyer receives confirmation of purchase of one GEM20 contract at its C-Last price, 96.965, and sale of one GEM22 contract at an imputed price of 96.955, equal to 96.965 minus the spread trade price of +1 tick (and conversely for the spread seller).

Butterfly = (+1 Leg1):(-2 Leg2):(1 Leg3)

To assign prices to the legs of a butterfly, CME Globex books Leg1 and Leg2 at their respective C-Last prices, then derives Leg3's price as:

$$\text{Leg3 price} = (\text{butterfly price}) \text{ minus } (\text{Leg1 price}) \text{ plus } (2 \times \text{Leg2 price})$$

Double Butterfly = (+1 Leg1):(-3 Leg2):(3 Leg3):(-1 Leg4)

CME Globex sets Leg1, Leg2, and Leg3 at their respective C-Last prices, then computes Leg4's price as:

$$\text{Leg4 price} = (\text{Leg1 price}) \text{ minus } (3 \times \text{Leg2 price}) \text{ plus } (3 \times \text{Leg3 price}) \text{ minus } (\text{double butterfly price})$$

Condor = (+1 Leg1):(-1 Leg2):(-1 Leg3):(1 Leg4)

Likewise, CME Globex books Leg1, Leg2, and Leg3 at their respective C-Last prices, and sets Leg4's price as:

$$\text{Leg4 price} = (\text{condor price}) \text{ minus } (\text{Leg1 price}) \text{ plus } (\text{Leg2 price}) \text{ plus } (\text{Leg3 price})$$

Examples 17

The Jun20-Sep20-Dec20 three-month butterfly trades at -1 tick. Assume C-Last prices for the Jun20 and Sep20 contracts are 96.98 and 96.97, respectively. CME Globex confirms to the butterfly buyer the purchase of one Jun20 at 96.98, the sale of two Sep20 at 96.97, and the purchase of one Dec20 at an imputed price of 96.95, equal to $-0.01 \text{ minus } 96.98 \text{ plus } (2 \times 96.97)$ (and conversely to the butterfly seller).

A Jun20-Jun21-Jun23-Jun24 one-year double butterfly trades at +0.5 ticks. Assume C-Last prices for the Jun20, Jun21, and Jun22 contracts are, respectively, 96.965, 96.995, and 96.97. CME Globex confirms to the double butterfly's buyer the purchase of one Jun20 at 96.965, the sale of three Jun21 at 96.995, the purchase of three Jun22 at 96.97, and the sale of one Jun23 contract at an imputed price of 96.885, equal to $(\text{Leg1 price}) \text{ minus } (3 \times \text{Leg2 price}) \text{ plus } (3 \times \text{Leg3 price}) \text{ minus } (\text{double butterfly price})$, or $96.965 \text{ minus } (3 \times 96.995) \text{ plus } (3 \times 96.97) \text{ minus } (-0.005)$.

For Pack calendar spreads, Pack butterflies, and Bundle spreads, price assignment proceeds in two stages. Prices of the spread's component Packs or Bundles are determined first, through a variant of the C-Last price mechanism. Then the resultant price of each component Pack or Bundle is apportioned to its constituent GE contracts on the basis of (a) each GE contract's net price change versus its latest daily settlement price and (b) the length of term to expiry for each GE contract relative to the other contracts in the Pack or Bundle, as described earlier. The first stage of the process requires a suitably modified definition of C-Last Price:

Pack/Bundle C-Last Price

The exchange establishes daily settlement prices only for futures contracts, not for Packs, nor for Bundles, nor for any other combinations or spreads. For this reason, unlike the C-Last price for a futures contract, the C-Last price for a Pack or Bundle makes no direct reference to any previous daily settlement price. Within any CME Globex trading session, therefore, the C-Last Price for any given Pack or Bundle is the most recent of:

- (a) latest CME Globex transaction price, or
- (b) CME Globex bid price that betters the bid side of the market, or
- (c) CME Globex asking price that betters the ask side of the market.

If there is no C-Last Price for the Pack or Bundle during the current CME Globex trading session, CME Globex calculates a synthetic price for it using the C-Last prices of its component GE contracts. For each such GE contract, CME Globex obtains the differential between the contract's C-Last Price and the contract's previous daily settlement price. It then computes the arithmetic average of these differentials among all component contracts in the Pack or Bundle. The average value gets rounded to the nearest $\frac{1}{4}$ tick, with tie values rounded towards zero. That is, an average net price change on the day of -2.125 ticks gets rounded up to -2 ticks, whereas an average of +5.625 ticks gets rounded down to +5.5 ticks.

For establishing the prices of the component Packs or Bundles in a Pack or Bundle spread, the ground rules are:

- Each leg is set to its C-Last price, except for the most deferred leg.
- For the most deferred leg, price is derived on the basis of the spread price and the price(s) assigned to the other leg(s).
- With prices thus established for all of the spread's legs, each Pack or Bundle price is then distributed to its respective member GE contracts, as described earlier.²⁰

²⁰ See "How Bundle and Pack Prices Become GE Contract Prices" on page 12.

Pack Spread = (+1 Leg1 Pack):(-1 Leg2 Pack)

CME Globex assigns to the Leg1 Pack its Pack C-Last price, defined above. The Leg2 Pack price is then imputed as the difference between the Leg1 Pack price and the Pack spread price –

Leg1 Pack price = Leg1 Pack C-Last price, and
 Leg2 Pack price = (Leg1 Pack price) minus (Pack spread price).

Pack Butterfly = (+1 Leg1 Pack):(-2 Leg2 Packs):(+1 Leg3 Pack)

The Packs on Leg1 and Leg2 are assigned their respective Pack C-Last prices. These and the Pack butterfly price are then used to impute the price of the Leg3 Pack –

Leg1 Pack price = Leg1 Pack C-Last price
 Leg2 Pack price = Leg2 Pack C-Last price
 Leg3 Pack price = (Pack butterfly price) minus (Leg1 Pack price) plus (2 x Leg2 Pack price)

Bundle Spread = (+1 Leg1 Bundle):(-1 Leg2 Bundle)

The Leg1 Bundle is set to its Bundle C-Last price, after which the Leg2 Bundle price is calculated as the difference between the Leg1 Bundle price and the Bundle spread price –

Leg1 Bundle price = Leg1 Bundle C-Last price
 Leg2 Bundle price = (Leg1 Bundle price) minus (Bundle spread price).

Examples 18

The Red-Green one-year Pack spread trades at +0.5 ticks. Because the GE contracts in the Red Pack are closer to final settlement, the Red Pack takes precedence in the price assignment hierarchy. It is automatically assigned its C-Last price, which is assumed to be +4 ticks. The imputed price for the Green Pack is +3.5 ticks, equal to the Leg1 price minus the spread price (+0.5 ticks). The buyer of the Pack spread receives buy confirmations for one each of the four GE contracts in the Red Pack, at prices equal to their respective previous daily settlement prices plus 4 ticks, and sell confirmations for each of the four GE contracts in the Green Pack, at prices equal to their respective previous daily settlement prices plus 3.5 ticks (and conversely for the Pack spread's seller).

The Red-Blue-Purple 2-year Pack butterfly trades at -0.5 ticks. To the Red and Blue Packs – the two legs comprising the nearest contract delivery months – CME Globex assigns their respective Pack C-Last prices. Suppose these are, respectively, +4 ticks and +2.5 ticks. The price of the Purple Pack – the leg comprising GE contracts with the most distant delivery months – is then derived as +0.5 ticks, equal to (butterfly price) minus (Red Pack price) plus (2 x Blue Pack price), or (-0.5 ticks) minus (+4 ticks) plus (2 x +2.5 ticks). For each Pack, the assigned price is then distributed to the Pack's constituent futures contracts in the usual manner.

Month-Pack Spread = +4 Leg1 GE:-1 Leg2 Pack

As mentioned earlier, the GE month-Pack spread is unique in mixing Pack combinations with outright positions in individual GE contracts. To distribute a month-Pack spread price to its legs, CME Globex always gives precedence to the singleton GE contract on Leg1. It is convenient to imagine the price attribution process working in four sequential steps:

- (a) Leg1 GE contract is set to its C-Last price.
- (b) Leg1 GE contract price is re-expressed in Pack-equivalent terms, as (C-Last price) minus (the contract's previous daily settlement price).
- (c) Leg2 Pack price is derived as (Leg1 Pack-equivalent price) minus (month-Pack spread price).
- (d) Leg2 Pack price, determined in Step (c), is translated into price levels for each of the Pack's member contracts, as described earlier.²¹

Example 19

For the buyer (and conversely for the seller) the Sep22 month-Pack spread comprises purchase of four Sep22 (GEU22) contracts and sale of one Gold-year Pack (one each of Dec22, Mar23, Jun23, and Sep23 contracts). Suppose it trades at +0.5 ticks. CME Globex apportions the spread price to the component contracts as follows:

- (a') The four GEU22 contracts on Leg1 are assigned the C-Last price. Assume it is 96.960.
- (b') The GEU22 price is re-expressed in Pack-equivalent terms. Assuming its previous daily settlement price is 96.945, its Pack-equivalent price is +1.5 ticks (equal to 96.960 minus 96.945).
- (c') For the Pack on Leg2, price is derived as +1 tick, equal to:
Leg1 Pack-equivalent price minus the month-Pack spread price, or (+1.5 ticks) minus (+0.5 ticks).
- (d') Each of the Pack's four component contracts is booked at its respective previous daily settlement price plus 1 tick.

CME Globex Trade Matching Algorithms For GE Futures²²

CME Globex matches every GE futures trade according to one of two matching algorithms, depending on the GE contract or spread or combination being transacted.²³ Before examining these, three bits of nomenclature are worth establishing –

Resting Order

A resting order is a posted actionable bid or offer at a given price for a given number of futures contracts (or for a given number of Bundles or Packs or other strategies standardized for trading on CME Globex).

Aggressor Order

An aggressor order is either a sell order that entails hitting resting bid orders, or a buy order that entails lifting resting offered orders.

TOP Order²⁴

The TOP order concept enters into trade matching only for outright buys/sells of GE contracts, calendar spreads, butterflies, double butterflies, condors, Pack spreads, and Pack butterflies.

At any given moment, for any given set of resting price levels and quantities in the CME Globex central limit order book, TOP order designation ensures that the first order to improve its side of the market will be filled before other resting orders on the same side of the market. Both the bid side and the offered side of the market may simultaneously have TOP orders; at any given moment, however, no more than one buy order and no more than one sell order can hold TOP designation.

An order achieves TOP status in one of two ways:

- as a newly entered resting order that either raises the best bid price or lowers the best offered price in the order book, or
- as a newly entered aggressor order that buys all resting offered quantities in the order book (or that sells out all resting bid quantities in the order book) and that leaves an unfilled remainder resting in the order book at a new best bid (or best offered) price.

²² The CME Globex trade matching algorithms described here incorporate implied pricing for all GE futures spreads and combinations except for Bundles, Bundle Spreads, and Month-Pack Spreads. For brevity's sake, the following discussion does not detail the process by which aggressor orders are matched to resting orders where the order book includes trade quantities arising from implied orders. For information on this subject, visit: <https://www.cmegroup.com/confluence/display/EPICSANDBOX/CME+Globex+Matching+Algorithms> and <https://www.cmegroup.com/confluence/display/EPICSANDBOX/Complex+Match+Example>

²³ For more information about CME Globex trade matching algorithms, visit: <https://www.cmegroup.com/confluence/display/EPICSANDBOX/CME+Globex+Matching+Algorithm+Steps>

For parameters that govern trade matching of GE futures outright or spreads or combinations of GE futures on CME Globes, the definitive resource is the CME Globex Product Reference Sheet at: <https://www.cmegroup.com/confluence/display/EPICSANDBOX/GCC+Product+Resources#>

²⁴ For more information about the rôle of TOP order functionality on CME Globex, visit: <https://www.cmegroup.com/confluence/display/EPICSANDBOX/CME+Globex+Matching+Algorithm+Steps#CMEGlobexMatchingAlgorithmSteps-TOP>

In both instances, the resultant displayed quantity must be greater than or equal to the minimum allowable TOP order size (for GE futures, one (1) contract).

An incumbent TOP order loses its TOP status if it is:

- superseded by a subsequent order that further improves the market, or
- satisfied up to the maximum allowable TOP order size (currently 49,999 contracts for any outright buy/sell, calendar spread, butterfly, or condor, and 8,000 contracts for any double butterfly, Pack spread, or Pack butterfly), or
- cancelled, or
- modified in any of the following ways:
 - an increase in its quantity, or
 - a change in its price, or
 - a change in its account number.

Importantly, TOP status can be awarded only to a newly arrived order. It is not automatically conveyed to the next best candidate in the order book, for example, when an incumbent TOP order is filled or cancelled or modified.²⁵

This means that, on either side of the market, there may not always be a TOP order. Consider for instance a newly arrived resting sell order that achieves TOP offer status. If it is subsequently cancelled, TOP status does not automatically pass to the next best resting offer already in the order book. Rather, TOP designation for the offered side of the market will be conferred only upon the next new sell order (either resting or aggressor) that improves the offered side.

Outrights, Calendar Spreads, Butterflies, Double Butterflies, Condors, Pack Spreads, and Pack Butterflies

...are matched according to the CME Globex Allocation (A) algorithm. The A algorithm distributes an incoming aggressor order pro rata to resting orders at the best price, with deference to the TOP order (if there is one) on the resting side of the market:

- (a) If there is a TOP order, the aggressor order is assigned to it first. If there is no TOP order, the process starts with Step (b).
- (b) Any unfilled portion of the aggressor order is matched to resting orders at the best price, on a pro rata basis. At any given resting price level, each resting order's pro-rated percentage is calculated by dividing its order quantity by the total quantity of all resting orders at that price level. Any unfilled portion of the aggressor order remaining after Step (a) is then multiplied by each resting order's pro-rated percentage.
- (c) The quantity allocated to each resting order equals the result obtained in Step (b) rounded down to the nearest integer number of contracts. For any resting order to which the rounded-down allocation would be less than two contracts, the quantity assigned is zero.
- (d) Any portion of the aggressor order that remains unfilled after application of Steps (b) and (c) is then assigned on a price-time priority (ie, "first-in-first-out", or FIFO) basis.

²⁵ In a similar spirit, any Good-until-Cancel (GTC) order that has earned TOP status during a given business week may retain its TOP designation at most until the ensuing weekend, at which time it loses its TOP status.

Exhibit 7 illustrates how the algorithm would work where a hypothetical aggressor order for sale of 633 contracts meets five resting orders at the best bid price, as described in Columns 1 and 2. ("Contract" is used loosely to signify either an individual GE futures contract or any of the eligible spreads or combinations listed above.) The five resting orders at the best price signify total depth at the best bid of 3,155 contracts –

- (a) The highest-priority resting order, for 150 contracts, holds TOP status. It gets matched first, in this case in its entirety (Column 3).
- (b) The remaining 483 contracts in the aggressor order are distributed pro rata across the 3,005 contracts remaining among the resting orders (Column 5). Each pro rata allocation is rounded down to the nearest integer number of contracts. Because the allocation to the second resting order in terms of time priority is less than two contracts, it is rounded down to zero (Column 6).
- (c) The remaining 2 contracts in the aggressor order are distributed FIFO. The second resting order in the time priority queue, for 5 contracts, absorbs this assignment (Column 8).

Column 9 summarizes the resultant distribution of the aggressor sell order among the resting bid orders.

**Exhibit 7 –Algorithm A – Pro Rata Allocation with TOP Price. Aggressor Order = 633 Contracts.
(Number of contracts, unless otherwise noted)**

(1) Resting Orders at Best Bid (Time Priority)	(2) Order Size	(3) TOP Allocation	(4) Resting Orders after TOP Allocation = (2)–(3)	(5) Pro Rata (Allocation Percentage)	(6) Pro Rata Allocation = (4)x(5)	(7) Resting Orders after Pro Rata Allocation = (4)–(6)	(8) FIFO Allocation	(9) Total Allocation = (3)+(6)+(8)
1 (TOP)	150	150						150
2	5		5	5 / 3005		5	2	2
3	1000		1000	1000 / 3005	160	840		160
4	500		500	500 / 3005	80	420		80
5	1500		1500	1500 / 3005	241	1259		241
Total		150			481		2	633

Source: CME Group

In any instance where the aggressor order quantity is greater than or equal to all resting quantity quoted at a given price level, CME Globex automatically bypasses steps (a) and (b) and directly matches all such resting orders on a FIFO basis. This exception improves efficiency and speed: Given that all such resting orders are certain to be matched, there is no reason to calculate pro rata assignment shares.

Packs, Bundles, Month-Pack Spreads, and Bundle Spreads

...are matched according to the "FIFO with LMM" (T) algorithm. Given that the exchange designates no lead market makers ("LMMs") in markets for GE futures or spreads or combinations, the T algorithm effectively operates as if it were a pure FIFO trade matching engine.²⁶

²⁶ In markets where the exchange recognizes lead market makers, the T algorithm first allocates any aggressor order to resting orders posted by such LMMs on the opposite side of the order book; thereafter, any remaining unmatched portion of the aggressor order gets matched to remaining resting orders on a FIFO basis.

Exhibit 8 illustrates with another hypothetical example. As in Exhibit 7 an aggressor sell order for 633 contracts meets five resting orders at the best bid. (Note that "contract" here denotes a combination – a Pack, Bundle, month-Pack spread, or Bundle spread – rather than an individual GE futures contract.) The two resting bids with highest time priority, for 150 contracts and five contracts, respectively, are fully satisfied. The resting bid with third highest time priority, for 1000 contracts is assigned the remaining 478 contracts in the aggressor order. The unfilled residual of this resting bid, 522 contracts, becomes first in the time-price priority queue.

**Exhibit 8 – Algorithm T – FIFO with LMM. Aggressor Order = 633 Contracts.
(Number of contracts, unless otherwise noted)**

Resting Orders at Best Bid (Time Priority)	Order Size	FIFO Allocation
1	150	150
2	5	5
3	1000	478
4	500	
5	1500	
Total		633

Source: CME Group

Daily Settlement Prices

The exchange bears sole responsibility not only for establishing final settlement prices of expiring contracts, but also for setting contract daily settlement prices, by reference to which the exchange determines daily marks-to-market.²⁷

Monthly Serials and the ¼-Tick-Eligible Quarterlies

For any Monthly Serial contract, and for any Quarterly contract for which the minimum price increment is ¼ tick, the daily settlement price is set as the (suitably rounded) volume weighted average price ("VWAP") of outright trades on CME Globex during the daily settlement period which, on any normal trading day, runs from 1:59pm to 2:00pm Chicago time.

Before any such VWAP gets put to work, the exchange rounds its raw value to the nearest integer multiple of the contract's minimum price increment. A tie value – a raw value that lands precisely at the midpoint of a minimum price increment – is resolved according to the "round toward zero" convention. For instance, for a Monthly Serial contract for which the minimum price increment is ½ tick, a raw VWAP value of 97.6525 would be rounded down to 97.650 (rather than up to 97.655).

If there are no trades during this one-minute period, then the contract daily settlement price is set equal to the midpoint between the lowest best bid price and the highest best offered price recorded during the settlement period, adjusted to incorporate bid and offered price information in calendar spreads among Monthly Serial contracts (plus the nearby Quarterly contract, if it is ¼-tick-eligible) and the nearest Quarterly contract that is not ¼-tick-eligible.

²⁷ See: <https://www.cmegroup.com/confluence/display/EPICSANDBOX/Eurodollar>

Quarterlies in the White, Red, and Green Delivery Years

For any contract among the nearest 12 Quarterly delivery months (excluding any nearby Quarterly contract for which the minimum price increment is ¼ tick), the daily settlement price is based on trading and quoting activity in both the outright market and certain spread markets on CME Globex during the daily settlement period, as follows:

- The initial daily settlement price value is the VWAP in outright trading.
- This initial value may be adjusted – but only within the range from the lowest best bid price to the highest best offered price observed in outright trading in the contract during the settlement period – so as to accommodate bid and offered prices observed in six specified calendar spreads and butterflies: 3-month, 6-month, 9-month, and 12-month calendar spreads, and 3-month and 12-month butterflies. The contract's daily settlement price is set equal to the solution value for which any such adjustment accommodates the greatest number of bid and offered prices in these spreads.

Quarterlies in the Blue, Gold, Purple, Orange, Pink, Silver, and Copper Delivery Years

For any contract among the most distant 28 Quarterly delivery months, the daily settlement price is based on two categories of price information observed on CME Globex during the daily settlement period:

- Bid and offered price indications in outright trading.
- Bid and offered price indications in calendar spreads or butterflies in which the contract serves as a component. These calendar spread and butterfly price indications are combined with daily settlement prices of GE contracts for which settlement prices already have been established, to create a set of imputed bid and offered prices for the contract at hand.

These data points – both outright bid and offered prices and estimated bid and offered prices – are combined to derive a best possible bid price and a best possible offered price in the contract. The contract's daily settlement price is set equal to the eligible price level (the integer multiple of the contract minimum price increment) that lies between the best possible bid and the best possible ask. If multiple eligible price levels are encompassed by the spread between the best possible bid and the best possible ask, then the daily settlement price is selected so that the net price change versus the contract's previous daily settlement price is as close as possible to the net change in the daily settlement price for the neighboring GE contract that is nearer to delivery.

Almost a Forward Rate, but Not Quite: Convexity Bias

A cousin of the GE futures contract is the three-month forward rate agreement ("FRA"), which trades widely in both bank funding markets and dealer-to-customer markets for treasury management products. Though superficially similar, the two are not identical. A salient distinction is that their market values respond differently to interest rate volatility; in consequence, GE futures contract rates systematically exceed the "true" forward interest rates at which the corresponding FRAs are priced.

Market practitioners refer to this phenomenon as the convexity bias in GE futures prices. The following passages describe how it comes about, and how market participants quantify it in practice.

Convexity in FRAs vs Linearity in GE Futures

Consider three simple examples in which the term structure of spot and forward interest rates is assumed to be flat at 4 percent per annum –

Example 20

A par FRA is agreed for a forward-starting 90-day rate of 4 percent, on a notional amount of \$1 million, for forward settlement two years hence, and for maturity two years and 90 days hence. Suppose that immediately thereafter all money market interest rates rise by one bp. If nothing else changes, then two years and 90 days from now this shift in market conditions will cost \$25 for the seller of the FRA (notionally, the future lender) and will reward the FRA buyer (notionally, the future borrower) by:

$$\$25 = \$1 \text{ million} \times (0.01 \text{ percent per year} / 100) \times (90 \text{ days} / 360 \text{ days per year})$$

Market convention, however, calls for the FRA's profit/loss to be reckoned in terms of its present value, not its value two years and 90 days from now. With money market interest rates assumed to have risen uniformly by one basis point to 4.01 percent per annum, each dollar due in two years and 90 days would be worth around 91.53 cents today. Accordingly, profit/loss on the FRA would be booked today at \$22.88 instead of \$25:

$$\$22.88 = \$25 \times 0.915337 = \$25 \times (1 + (4.01 \text{ percent}/100))^{-2.25}$$

Example 21

The same applies to an FRA for any term to forward settlement. Thus, for a 90-day FRA for forward settlement nine years hence, and which is otherwise identical to the FRA in Example 20, an ensuing one bp rise in interest rates to 4.01 percent would also translate to a forward profit/loss of \$25, to be recognized nine years and 90 days in the future. Discounted to present at the new prevailing market rate of 4.01 percent per annum, the profit/loss booked today would be:

$$\$17.38 = \$25 \times 0.695113 = \$25 \times (1 + (4.01 \text{ percent}/100))^{-9.25}$$

Example 22

Now compare the FRAs in Examples 20 and 21 to their corresponding GE futures.²⁸ With all money market interest rates assumed to start at 4 percent per annum, a naïve but seemingly reasonable assumption is that GE futures for delivery either two years hence or nine years hence should be priced at 96.000, equal to 100 minus 4 percent. In both cases, moreover, a uniform one-basis-point rise in market expectations of future interest rates presumably would reduce both contract prices to 95.990, equal to 100 minus 4.01 percent. Most important, the ensuing daily marks-to-market for both contracts presumably would equal \$25 because, unlike their corresponding FRAs, GE futures are defined so that *each basis point of contract interest is always worth \$25, irrespective of when the futures contract expires.*²⁹

Because the interest rate sensitivity of each GE contract is \$25 per basis point, always and everywhere, its response to interest rate changes is described as "linear." Because the interest rate sensitivity of the FRA depends on both prevailing interest rate levels and the FRA's term to expiry, the relationship between changes in its value and changes in its reference interest rate is described as "convex."

A key consequence of this difference is that GE futures are not, in fact, priced as conjectured in Example 22. How they actually get priced is revealed in the dynamics of their cash-futures basis.

Volatility and the GE cash-futures basis

GE cash-futures spread positions are sensitive to interest rate volatility in a peculiar way: In the absence of any correction for the structural differential between linear pricing of GE futures versus convex pricing of the adjacent FRAs, any movement in market interest rates tends to benefit the buyer of the basis position (the notional future lender in the FRA and seller of GE futures) to the detriment of the basis seller (the notional future borrower in the FRA and buyer of GE futures). The next two examples illustrate.

Example 23

Return to the long-dated FRA in Example 21, and assume its notional principal amount is \$100 million instead of \$1 million. The profit/loss to be realized nine years and 90 days from now will be \$2,500 for each basis point of change in the FRA's reference interest rate. For the purpose of reckoning the profit/loss to be booked today, the DV01 of the FRA is the present value of \$2,500 to be received nine years and 90 days in the future:

$$\$1,738 \approx \$2,500 \times 0.695113$$

²⁸ If the circumstances in Example 22 take place on the eve of final settlement of the nearby Quarterly GE contract, then these two GE futures would be the First Green and the First Copper, respectively. See "Delivery Month and Last Trading Day" on pages 6-7.

²⁹ See "Contract Size and Minimum Price Increment" on pages 9-10.

To buy the corresponding cash-futures basis spread, one would sell \$100 mln notional of the FRA as the cash leg and sell 70 GE futures for the first Copper delivery month.³⁰ The spread ratio of 70 futures is chosen to bring the cash-futures spread as close as possible to (local) DV01-neutrality:

$$\begin{aligned} \text{GE leg DV01} &= \text{FRA leg DV01} \\ 70 \text{ contracts} \times (\$25 \text{ per bp per contract}) &= \$1,750 \text{ per bp} \approx \$1,738 \text{ per bp} \end{aligned}$$

Now assume all market interest rates rise 10 bps, to 4.10 percent. The basis spread position earns a profit of \$261, with the FRA component losing \$17,239, while the short GE component earns \$17,500 –

$$\begin{aligned} \text{Loss on FRA position} &= -\$17,239 \\ &= +10 \text{ bps} \times (-\$2,500 \text{ per bp}) \times (1 + (4.10 \text{ percent}/100))^{-9.25} \end{aligned}$$

$$\begin{aligned} \text{Earnings on short position in GE futures} &= \$17,500 \\ &= (-10 \text{ GE price ticks}) \times (\$25 \text{ per tick per contract}) \times (-70 \text{ contracts}) \end{aligned}$$

Alternatively, assume all interest rates fall 10 bps, to 3.90 percent. Once again, the basis spread earns a profit, this time \$49. Calculations similar to those above show the FRA position has earned \$17,549, while the short GE position has lost \$17,500.

The "peculiar" feature of the trade is obvious. In the absence of any correction for the difference between linear pricing of GE futures and convex pricing of FRAs, the buyer of the cash-futures basis enjoys a profit, and the seller bears a loss, irrespective of whether market interest rates rise or fall.

Example 24

To appreciate the implications in richer detail, consider the following hypothetical but plausible set of assumptions:

It's afternoon on Monday before the third Wednesday of March. Earlier in the day, the March GE contract came to final settlement.

- The daily settlement price for the first Copper GE contract is 94.355, signifying a contract interest rate of 5.645 percent per annum for forward settlement in mid-June 9-1/4 years hence, and for maturity in mid-September 9-1/2 years hence.
- The term of the hypothetical forward-starting bank funding deposit corresponding to the first Copper GE contract interest rate is 92 days.³¹ This implies that, on a hypothetical \$100 million bank placement, a one-basis-point change in the contract interest rate would create a forward profit/loss of \$2,555.56, to be recognized 9-1/2 years hence.
- Conditions in money markets and in markets for short-term interest rate derivatives jointly indicate that any profit or loss to be realized 9-1/2 years hence should be discounted to present value at a rate of 3.76 percent per annum.

30 Market practitioners familiar with cash-futures basis spreads in, eg, Treasury Note and Bond futures or grain futures will be accustomed to thinking of a long basis position as "long cash plus short futures" – ie, as comprising a long position in the futures contract's reference commodity plus a short position in futures. By comparison, the definition of the GE futures basis – the sale of a FRA combined with sale of the corresponding GE futures – may look odd.

The explanation is that purchases and sales of FRAs are conventionally defined in terms of interest rate exposure, not asset price exposure. The FRA is a contract for a cash payment at maturity based on the difference between a future spot interest rate and the FRA contract rate. At the FRA's expiration, if the spot interest rate is higher than the FRA contract rate, then the FRA "seller" pays the "buyer" the difference between the FRA contract rate and the spot rate (multiplied by the FRA's notional principal amount). Conversely, if the spot rate is lower than the FRA contract rate, the "buyer" pays the "seller" the appropriate amount upon the FRA's expiry. Thus, the roles of "buyer" and "seller" in the FRA are opposite to the respective roles performed by the buyer and seller of a GE futures contract. See "Before Futures Expiration – Contract Price and Contract Interest Rate" on page 5.

31 A safe assumption: The three-month interval starting on the third Wednesday of June most frequently spans 92 days.

For the par FRA corresponding to the first Copper GE contract, these assumptions imply that the DV01 for reckoning the profit/loss to be booked today is:

$$\$1,780 = \$2,555.56 \times 0.704231 = \$2,555.56 \times (1 + (3.76 \text{ percent}/100))^{-9.5}$$

Thus, a cash-futures basis position built on \$100 million notional of the FRA would require a countervailing position of 72 first Copper GE futures:

$$72 \text{ GE contracts} \approx (\$1,780 \text{ per bp}) / (\$25 \text{ per bp per GE contract})$$

What sort of profit/loss profile might this cash-futures basis spread position generate? To illustrate, let's add two more sets of assumptions about the distribution of money market interest rate dynamics:

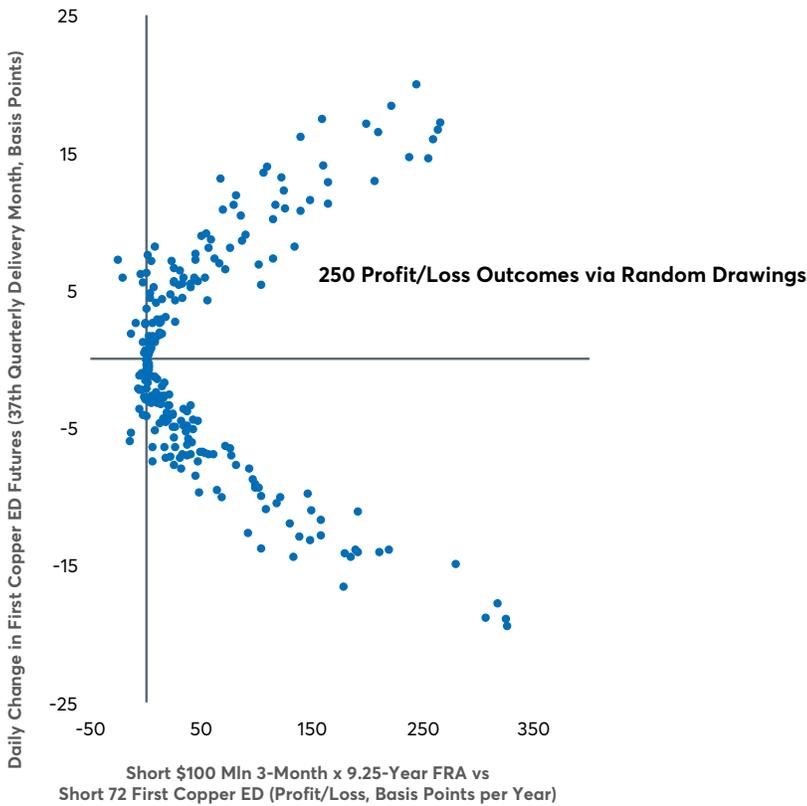
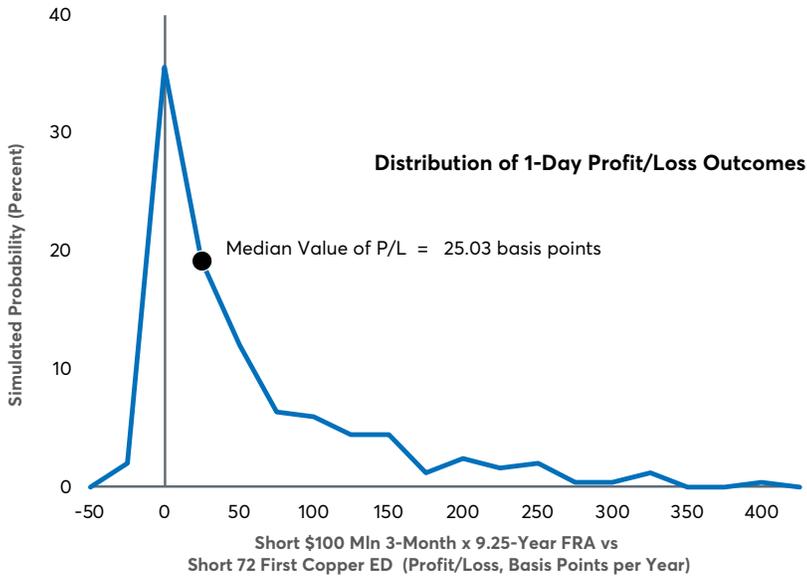
- Standard deviation of daily changes in the 9-1/4 year three-month FRA rate = 8.13 bps/yr.
Standard deviation of daily changes in the 9-1/2 year term discount rate = 7.32 bps/yr.
Correlation between daily changes in these two rates = 0.89 (on a scale of -1 to +1).
- The joint distribution of daily changes in these two interest rates is bivariate Gaussian, with zero mean for each interest rate, and with variance-covariance matrix as described above.

The upper panel of Exhibit 9 depicts an estimate of the distribution of one-day profit/loss outcomes, based on 250 random draws from the assumed distribution.

The lower panel of Exhibit 9 evidences the "peculiarity" described in Examples 23 and 24: A long position in the cash-futures basis (as before, the sale of the FRA combined with a delta-equivalent short position in GE futures) seldom loses money; here, it produces a net loss on just 15 draws out of every 100, and in no event is the net loss large.

From the long basis holder's standpoint, the basis spread clearly rewards volatility – the higher the volatility, the more lavish the reward – irrespective of whether volatility produces a rise or a fall in the forward rate from which the FRA and the GE futures contract derive. The point is dramatized by the long-tailed skew in the marginal distribution of profit/loss, shown in Exhibit 9's upper panel.

Exhibit 9 – GE Futures Convexity Bias via Simulation
(See Example 24 for details of simulation.)



Examples 23 and 24 address the same fundamental question: If the cash-futures basis spread is slanted so sharply in favor of anyone who acquires interest rate volatility exposure by buying the basis, then what would motivate anyone to sell volatility exposure by selling the basis?

Convexity bias via leveling the playing field

The answer is that a market participant will sell the basis if she can buy the GE futures at a discount sufficient to give her at least an even shot at making a profit.

This phenomenon is well known to market practitioners. For many years, GE futures have traded at market prices that incorporate a systematic discount, precisely so that prospective returns get evened up between the long and short sides of the cash-futures basis.

The upper panel in Exhibit 9 is suggestive of how the size of this discount might be determined. If there were no corrective discount to the GE futures price (as presumed in Example 24), then the median point in the estimated distribution of profits accruing to the long holder of the cash-futures basis would be approximately 25 bps per annum. In other words, spot and forward interest rate volatilities are such that a price reduction of 0.25 price points (a contract interest rate increase of 25 bps) applied to the first Copper GE futures contract would level the playing field, giving both the buyer and the seller of the basis spread approximately equal odds of making a gain or loss.

If market participants make full and efficient use of the information available to them, including information about interest rate volatility, then it is reasonable to assume that the corrective price discount for the convexity bias is already built into observed GE futures prices:

$$\text{Observed GE contract interest rate} = \text{"True" forward interest rate} + \text{Correction for convexity bias}$$

In the context of Example 24, with the appropriate correction for convexity bias for the first Copper GE contract estimated to be 0.25 percent per annum, the "true" (FRA-equivalent) forward interest rate implied by the futures contract daily settlement price is around 5.395 percent:

$$5.645 \text{ percent} = 5.395 \text{ percent} + 0.25 \text{ percent}$$

Determinants and a rule of thumb

Though it would be cumbersome, one could perform the same exercise as in Example 24 and Exhibit 9 for every GE contract delivery month. What one would find is that the magnitude of correction for any GE futures contract generally grows larger as:

- (1) the term from today to the GE contract delivery month lengthens (as suggested, eg, by comparison of Examples 20 and 21); or
- (2) volatility increases in the forward-starting three-month interest rate that the GE contract references; or
- (3) volatility increases in the spot-starting term interest rate used to establish the present value of changes in the GE contract's forward-starting interest rate; or
- (4) the degree of correlation increases between movements in the forward interest rate (2) and movements in the spot interest rate used for discounting to present (3).

Of these four factors, the two that are most salient are the level of volatility in the forward-starting three-month interest rate, and the time span to futures contract expiry.

A popular rule of thumb³² elegantly reflects their interrelationship:

$$\text{Convexity bias} = (\sigma^2/2) \times t \times T$$

where

t = term from now to third Wednesday of the GE futures contract delivery month (in years)

T = t + 3 months (ie, t + 0.25 years)

σ^2 = variance of the forward-starting three-month interest rate for settlement on third Wednesday of the GE futures delivery month.

The upper panel of Exhibit 10 depicts approximately how this rule of thumb would have applied to Quarterly GE futures contracts on 19 November 2018. In the upper panel, the light blue line traces the term structure of raw GE futures contract interest rates (at each point, 100 minus the futures contract price), and the dark blue line is the corresponding term structure of "true" FRA-equivalent forward interest rates (at each point, the raw GE futures contract rate minus the estimated correction for the futures contract's convexity bias). The lower panel shows the spread between the two term structures, ie, the magnitude of correction for convexity bias.³³

For instance, let σ be the volatility of the forward-starting three-month interest rate that corresponds to the first Copper GE contract (GEZ27, for delivery in December 2027). Assume it is 69 basis points per annum (approximately 4.3 basis points per day, equal to (69 bps/yr) / (252 business days/yr)^{1/2}). Then the values required for evaluation of the rule of thumb are:

$$\sigma^2 = 0.00004761 = 0.0069^2$$

t = 9.25 years

T = 9.5 years

The estimated magnitude of convexity bias is roughly 20.9 basis points per annum:

$$0.002092 = (0.00004761 / 2) \times 9.25 \times 9.5$$

32 Formulated by John Hull and Alan White. See John Hull, "Technical Note No. 1, Options, Futures, and Other Derivatives – Convexity Adjustments to Eurodollar Futures", nd, available at: <http://www-2.rotman.utoronto.ca/~hull/technicalnotes/TechnicalNote1.pdf>

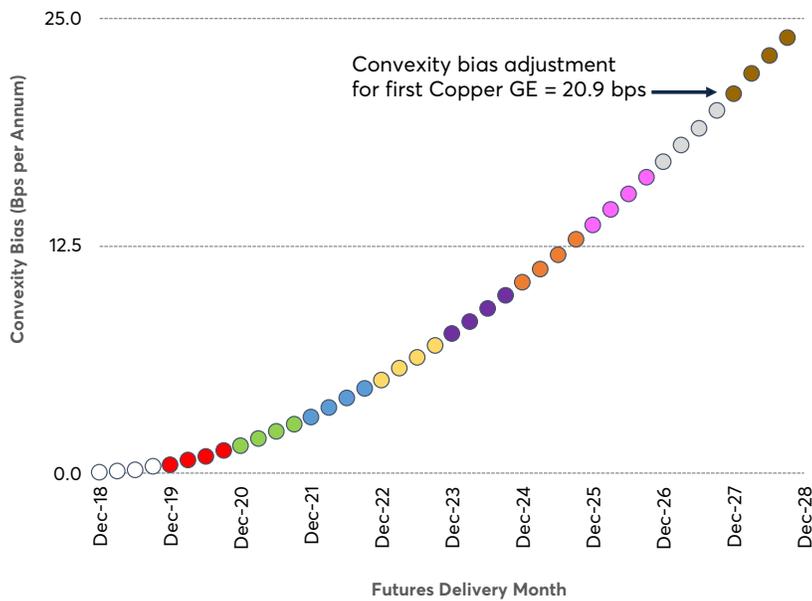
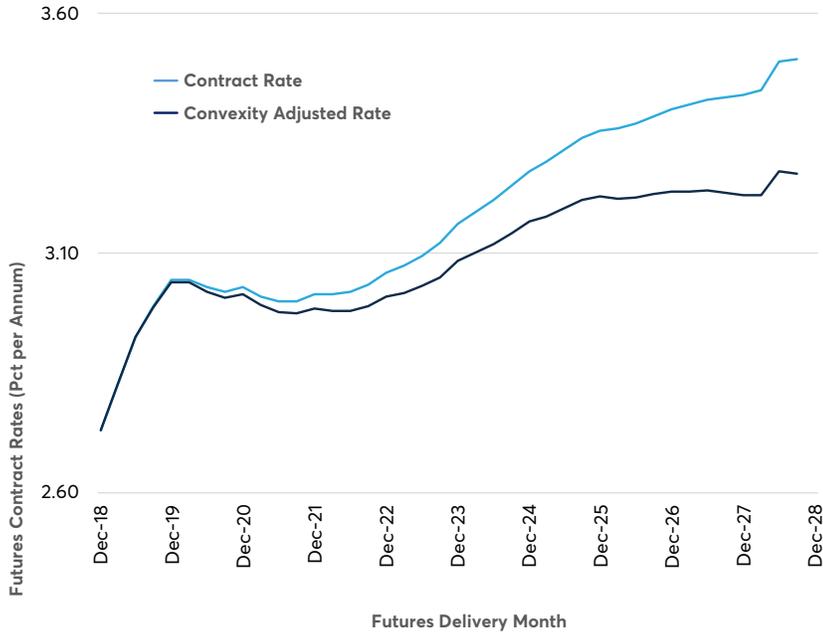
Until the mid-2000s, the spot-starting term interest rate used for reckoning present values of future cash flows would have been presumed to be based on long-dated exposure to (the predecessor of) three-month US dollar ICE LIBOR[®]. Likewise, the correlation referenced above would have been assumed to apply to dynamics between the forward-starting three-month LIBOR and the spot-starting term interest rate based on long-dated exposure to three-month LIBOR.

Since the mid-2000s, the consensus among market practitioners has shifted, such that (a) the spot-starting term interest rate employed for discounting future amounts to present value is assumed to be associated with a hypothetical overnight index swap (OIS) based upon the daily effective federal funds rate and (b) the correlation coefficient is assumed to characterize the distributional relationship between the forward-starting three-month LIBOR and the spot-starting OIS that spans the term to the forward start date.

With appropriate modifications to the underlying mathematics, the framework that supports this rule of thumb can accommodate either discounting basis. See, eg, Fabio Mercurio, "The Present of Futures: Valuing Eurodollar-Futures Convexity Adjustments in a Multi-Curve World", 16 June 2017, available at: <https://ssrn.com/abstract=2987832>, or Fabio Mercurio, "The Present of Futures", Risk.net, March 2018. See also John Hull and Alan White, "LIBOR vs OIS: The Derivatives Discounting Dilemma", April 2013, available at: <https://www-2.rotman.utoronto.ca/~hull/DownloadablePublications/LIBORvsOIS.pdf>

33 The methodology underlying these estimates is based on the Hull and White formulation referenced in Footnote 31. For details, see Bloomberg, SWPM <GO> <HELP> <White Papers> <Eurodollar Futures Convexity Adjustment Calculation>

Exhibit 10 – GE Futures Convexity Bias: 19 November 2018
Upper Panel: GE Contract Rates with and without Adjustment for Convexity Bias
Lower Panel: Size of Convexity Bias



Source: Bloomberg LP

In a nutshell

- The GE futures contract is defined so that its response to interest rate changes is linear. The contract's DV01 with respect to its underlying forward three-month interest rate is always \$25.
- FRAs are priced and marked so that their response to interest rate changes is convex. The DV01 of a \$1 million FRA with respect to its underlying forward three-month interest rate is the present value of approximately³⁴ \$25 per basis point of change in the rate.
- Because the GE futures buyer does not enjoy the benefit of being an owner of convexity, GE futures are systematically priced at a discount relative to the (hypothetical) contract price that would be determined as 100 minus the corresponding FRA forward rate.
- In other words, at fair value, GE futures contract rates are systematically higher than the corresponding FRA forward interest rates. The difference (GE futures contract rate minus FRA forward rate) is known as the convexity bias correction:
- GE contract rate = "true" FRA forward rate + convexity bias correction
- The convexity bias correction is a positive function of interest rate volatility and term to futures expiry.

One-Month Eurodollar Futures

One-Month Eurodollar (GLB) futures, introduced in April 1990, are just like their older GE siblings, except shorter and more concentrated. Exhibit 11 summarizes contract features.

Exhibit 11 — One-Month Eurodollar Futures Contract Specifications

(All times of day are Chicago time unless otherwise noted.)

Contract Unit	\$2,500 x contract IMM Index
Price Basis	Contract IMM Index = 100 minus contract interest rate Contract <i>interest rate</i> = interest rate per annum on one-month unsecured bank borrowing, for spot settlement on 3rd Wednesday of contract Delivery Month. (Eg, a contract price of 97.45 IMM Index points signifies a contract interest rate of 2.55 percent per annum.)
Price Increments	¼ of one bp (0.0025 IMM Index points) = \$6.25 per contract
Delivery Months	Nearest 24 calendar months
Last Trading Day	2nd London bank business day before 3rd Wednesday of Delivery Month
Delivery Standard	One-month US dollar ICE LIBOR® as set on Last Trading Day for spot (T+2) settlement on 3rd Wednesday of Delivery Month, rounded to nearest 1/100th of one bp
Delivery Method	Cash settlement by mark-to-market to Final Settlement Price. <i>Final Settlement Price</i> = Price Basis evaluated at Delivery Standard. Final settlement occurs on Last Trading Day.
Trading Hours	5pm to 4pm, Sun-Fri. Trading in expiring contracts terminates at 11am London time on Last Trading Day.
Position Accountability	5,000+ contracts
Reportable Positions	100+ contracts
Block Trade Minimum Sizes	ATH (4pm to 12am): 100+ contracts ETH (12am to 7am CT): 200+ contracts RTH (7am to 4pm CT): 400+ contracts
Designated Contract Market	CME One-Month Eurodollar futures are listed for trading on, and subject to the rules and regulations of, CME.

Source: CME Group

³⁴ *Reminder: The FRA's DV01 is the present value of exactly \$25 per basis point only if the FRA's forward-starting three-month interval spans 90 days. Otherwise, the FRA's DV01 is the present value of some other dollar amount per basis point. See "The Parable of the \$1 Million" on pages 10-11.*

³⁵ ICE LIBOR® is a registered trademark of Intercontinental Exchange Holdings, Inc. and is used under license.

Contract Unit and Price Basis

The GLB contract price is gauged in terms of the IMM index, 100 minus the contract interest rate. The contract interest rate is the rate per annum on a hypothetical unsecured bank funding deposit for spot settlement on the third Wednesday of the contract delivery month, with term to maturity of one month, for which interest accrues on an actual/360-day basis, and for which each basis point per annum of the corresponding rate of interest is worth \$25.

Delivery Month and Last Trading Day

The exchange typically lists contracts for delivery in each of the nearest 24 calendar months, ie, 12 consecutive monthly contracts in each of the White and Red delivery years. Identical to GE futures, each GLB contract ceases trading and comes to final settlement on the second London business day (typically the Monday) before the third Wednesday of its delivery month. Just prior to termination of trading in the expiring nearby contract (on the Monday before the third Wednesday of the month), a new twenty-fourth contract delivery month is listed for trading when the CME Globex session opens on the preceding Sunday afternoon.

Final Settlement Price

An expiring GLB contract's final settlement price is determined as 100 minus one-month US dollar ICE LIBOR® on the second London bank business day preceding the third Wednesday of the contract delivery month. Before applying the ICE LIBOR® value to establish the final settlement price, the Exchange rounds it from five decimal places to four. A tie value, ie, any published one-month ICE LIBOR® value ending in 0.00005, gets rounded up to four decimal places.

Contract Size and Price Increments

The sole determinant of GLB contract notional size is the requirement that each price tick – each basis point per annum of contract interest rate exposure (each 0.01 contract IMM Index points) -- is worth \$25. As with GE futures, the definition of GLB's tick size automatically implies the size of the notional one-month bank funding placement corresponding to it, but the converse does not hold. As Exhibit 12 demonstrates:

- The principal amount of the funding transaction would be exactly \$3 million if the length of the corresponding one-month term were exactly 30 days, a fact pattern which applies in one of every three months.
- Most common, occurring in roughly 11 months out of every two years, is where the one-month term spans 33 days; in such instances, the notional bank placement's size would have to be approximately \$2.73 million to keep the dollar value of each basis point of interest fixed at \$25.
- More generally, implied sizes of the GLB contract's notional underlying bank placement range from more than \$3.2 million (where the one-month term lasts 28 days) down to approximately \$2.6 million (where the one-month term runs to 34 days).

The minimum price movement for trading GLB futures is always ¼ tick, or one quarter of one interest rate bp (0.0025 contract IMM Index points), equal to \$6.25 per contract.

Exhibit 12 — Day Counts and Notional Bank Funding Placement Sizes for US Dollar 1-Month Term Wholesale London Bank Deposits for Settlement on 3rd Wednesday of Month, Oct 1977 through Sep 2018

Day Count of 3-Month Bank Deposit	Deposit Size Required for DV01 to Equal \$25 (Dollars)	Number of Occurrences	Pct Share of Occurrences
28	3,214,285.71	31	6
29	3,103,448.28	10	2
30	3,000,000.00	164	33
31		0	0
32		0	0
33	2,727,272.73	224	46
34	2,647,058.82	63	13

Source: CME Group

Spreads and Combinations

In addition to outright trades in GLB futures, CME Globex permits market participants to trade calendar spreads and butterflies (broadly similar to the GE calendar spreads and butterflies described on page 13-14) as well as GLB-GE inter-market spreads. In all cases, the minimum price increment for quoting and trading is ¼ tick (0.0025 contract price points) per spread.

Trade Matching for GLB Futures

For all GLB contracts, spreads, and combinations, CME Globex matches buyers and sellers with the Pro Rata Allocation with TOP Price (or A) algorithm described on pages 23-24.

Appendix: Day Count Conventions for Money Market Transactions

Unless otherwise specified, an overnight US dollar bank funding transaction quoted "today" is understood to be for same-day (T+0) settlement, provided that "today" is both a London bank business day and a New York bank business day.

For any term to maturity other than overnight, and unless otherwise specified, a US dollar bank funding transaction is assumed to be quoted for spot (T+2) settlement. That is, a bank placement agreed "today" is understood to settle two London bank business days after "today," provided that the settlement date is also a New York bank business day. If it is not, then the deposit is assumed to settle on the next following day that is both a London bank business day and a New York bank business day ("business day").

For overnight or 1-week terms to maturity

The maturity date is understood to be the first business day that falls at least one or seven days, respectively, from the settlement date.

For monthly (1-month to 12-month) terms to maturity

As a general principle, the maturity date occurs within the month of maturity, on the first business day on or after the date that matches the settlement date. For instance, a 1-month deposit for settlement on 5 March is understood to mature on 5 April, where both dates are business days. Specifics vary, however, according to the circumstances --

Standard Procedure: Next Business Day

If the corresponding date in the month of maturity is not a business day, then the maturity date is the next following date that is a business day.

Exception 1: Modified Next Business Day ("Modified Following")

Where the Standard Procedure would result in the maturity date getting pushed from the month of maturity into the next following calendar month, then the maturity date is assigned instead to the first preceding date that is a business day.

Exception 2: End-End

If the deposit's settlement date happens to occur on the final business day of a calendar month, then the maturity date is always set to the last business day of the month of maturity, rather than to the corresponding date within the month of maturity. For example, a four-month term placement that settles on 30 November is understood to mature on 31 March, not on 30 March (where we assume, for convenience, that 30 November, 30 March, and 31 March all are business days).

Exhibit A illustrates how these procedures and exceptions would have interacted when applied in determining dates for one-month bank funding placements made in late January 2015.

Exhibit A – Settlement Date and Maturity Date Examples for 1-Month Bank Funding Placements, Jan 2015

Transaction Date	Settlement Date (T+2)	Maturity Date	Applicable Term to Maturity Rule
Thu, 22 Jan	Mon, 26 Jan	Thu, 26 Feb	Standard
Fri, 23 Jan	Tue, 27 Jan	Fri, 27 Feb	Standard
Mon, 26 Jan	Wed, 28 Jan	Fri, 27 Feb	Modified Following
Tue, 27 Jan	Thu, 29 Jan	Fri, 27 Feb	Modified Following
Wed, 28 Jan	Fri, 30 Jan	Fri, 27 Feb	End-End
Thu, 29 Jan	Mon, 2 Feb	Mon, 2 Mar	Standard
Fri, 30 Jan	Tue, 3 Feb	Wed, 3 Mar	Standard

Source: CME Group

Resources

CME Rulebook Chapter 452 -- Three-Month Eurodollar Futures Terms and Conditions
www.cmegroup.com/rulebook/CME/V/450/452/

CME Rulebook Chapter 453 -- One-Month Eurodollar Futures Terms and Conditions
www.cmegroup.com/rulebook/CME/V/450/453/

ICE Benchmark Administration Ltd – ICE LIBOR®
<https://www.theice.com/iba/libor>

UK Bank Holidays
www.direct.gov.uk/en/Governmentcitizensandrights/LivingintheUK/DG_073741

US Bank Holidays
www.newyorkfed.org/aboutthefed/holiday_schedule.html

Galen Burghardt, The Eurodollar Futures and Options Handbook
 McGraw-Hill, 2003



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