CME GROUP-MSRI

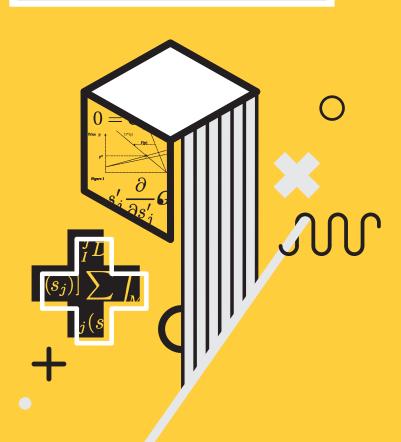
PRIZE *in* INNOVATIVE QUANTITATIVE APPLICATIONS

MODERATED SEMINAR

— and —

AWARD LUNCHEON FEB 2 **2017**

CHICAGO, ILLINOIS



CME GROUP-MSRI PRIZE IN INNOVATIVE QUANTITATIVE APPLICATIONS

Awarded annually, the CME Group-MSRI Prize in Innovative Quantitative Applications rewards exemplary work in the field of mathematical sciences and recognizes the vital impact of quantitative research and its application to shaping global financial markets.

CME Group, the world's leading and most diverse derivatives marketplace, launched the CME Center for Innovation in 2003. The Center creates and sponsors thought-provoking original programming that identifies and fosters examples of significant innovation and creative thinking across multiple industries. Each of the programs aims to explore the forces that drive innovation and showcase their application to a broad and diverse audience.

The Mathematical Sciences Research Institute (MSRI) partners with CME Group in this initiative. Based in Berkeley, California, MSRI exists to further mathematical research through broadly based programs in the mathematical sciences and closely related activities. Each year, approximately 2,000 mathematicians visit the Institute, which is funded primarily by the National Science Foundation, with additional support from other government agencies, private foundations, individual and corporate donors, and academic sponsors.



2016 PRIZE RECIPIENT

ROBERT WILSON

ROBERT WILSON is the Adams Distinguished Professor of Management, Emeritus, at Stanford Graduate School of Business. His research focuses on game theory and its applications in economics, such as design of auctions and related markets. A theme of his joint work with David Kreps was dynamic effects of different information among economic agents, and with Srihari Govindan, foundational studies of axiomatic criteria for selecting among equilibria. His recent research studies sufficient conditions for repeated interactions to sustain cooperative behaviors.

He has co-authored some of the basic studies of reputational effects in predatory pricing, price wars and other competitive battles. He has contributed to auction designs and competitive bidding strategies in the oil, communication and power industries. His work includes studies of wage bargaining and strikes and, in legal contexts, settlement negotiations.

Wilson has been at Stanford Business School since 1964. He is a member of the National Academy of Sciences, a distinguished fellow of the American Economic Association, former president of the Econometric Society, recipient of honorary degrees from the Norwegian School of Economics and the University of Chicago, and winner of the Foundation BBVA Frontiers of Knowledge prize in finance and economics. His book on nonlinear pricing won the University of Chicago's Melamed prize.

MODERATED SEMINAR

FRONTIERS OF GAME-THEORETIC APPLICATIONS IN ECONOMICS

WELCOME

LEO MELAMED

Chairman Emeritus, CME Group

MODERATOR

DAVID EISENBUD

Director, Mathematical Sciences Research Institute (MSRI)

SEMINAR PART ONE

CONTRIBUTIONS TO GAME THEORY IN ECONOMICS

PANELISTS

DREW FUDENBERG

Professor of Economics, Massachusetts Institute of Technology

SRIHARI GOVINDAN

Professor, Department of Economics, University of Rochester

ROGER MYERSON

Nobel Laureate in Economic Sciences (2007), Glen A. Lloyd Distinguished Service Professor of Economics, University of Chicago SEMINAR PART TWO

VIEWS ON THE WORK OF ROBERT WILSON, BY HIS FORMER STUDENTS

PANELISTS

ALVIN ROTH

Nobel Laureate in Economic Sciences (2012), Craig and Susan McCaw Professor of Economics, Department of Economics, Stanford University

PAUL MILGROM

Shirley R. and Leonard W. Ely Jr. Professor of Humanities and Sciences, Economics Department, Stanford University

BENGT HOLMSTRÖM

Nobel Laureate in Economic Sciences (2016), Paul A. Samuelson Professor of Economics, Massachusetts Institute of Technology and 2013 Recipient of the CME Group-MSRI Prize in Innovative Quantitative Applications

$$\frac{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}}{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}} = E\{\sum_{I} D_{I}^{j} \mid p \& \sum_{\ell} s_{\ell} = b_{\circ}\}. \quad 0 = G_{j} - s_{j}^{\prime} \frac{1}{\partial s_{j}^{\prime}} \frac{1}{\partial s$$

LUNCHEON AND AWARD CEREMONY

APPRECIATION OF THE LIFE AND WORK OF ROBERT WILSON

PHILIP RENY

The Hugo F. Sonnenschein Distinguished Service Professor in Economics and the College, University of Chicago

KFYNOTF ADDRESS

ROBERT WILSON

Adams Distinguished Professor of Management, Emeritus Stanford Graduate School of Business

PRESENTATION OF THE CME GROUP-MSRI PRIZE IN INNOVATIVE QUANTITATIVE APPLICATIONS

LEO MELAMED

Chairman Emeritus, CME Group

DAVID EISENBUD

Director, Mathematical Sciences Research Institute (MSRI)

PROGRAM CONCLUSION

$$\frac{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}}{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}} = E\{\sum_{I} D_{I}^{j} \mid p \& \sum_{\ell} s_{\ell} = b_{o}\}$$

66

I'M GLAD TO RECEIVE THIS AWARD FOR MY WORK APPLYING GAME THEORY TO ECONOMICS. IT'S A POWERFUL TOOL FOR DETAILED STUDIES OF DYNAMIC INTERACTIONS AFFECTED BY INFORMATIONAL DIFFERENCES.
THE DIVERSE APPLICATIONS INCLUDE BILATERAL BARGAINING, MULTILATERAL MARKETS AND AUCTIONS, AND COMPETITION AMONG FIRMS. ITS USE HAS ENABLED MORE REALISTIC THEORY, EMPIRICAL STUDIES AND PRACTICAL APPLICATIONS TO MARKET DESIGN.

- ROBERT WILSON

Adams Distinguished Professor of Management, Emeritus, Stanford Graduate School of Business

$$\frac{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}}{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}} = E\{\sum_{I} D_{I}^{j} \mid p \& \sum_{\ell} s_{\ell} = b_{\circ}\}.$$

$$0 = G_j - s_j' rac{\partial s_j'}{\partial s_I'} G_{j_p}$$

WELCOME ADDRESS

LEO MELAMED

Chairman Emeritus, CME Group



Leo Melamed has served as a member of the CME Group board of directors since 2001. He is the founder of financial futures and was instrumental in the creation of the CME Globex platform. He has served as CME Chairman Emeritus since 1997 and Chairman of the company Strategic Steering Committee since 2001. He served as chairman of the board from 1968 until 1973. He was founding Chairman of the International Monetary Market (IMM) from 1972 until its merger with CME in 1976, and

then CME Chairman until 1977. Melamed served as a special advisor to the company in the role of Special Counsel to the board from 1977 to 1985 and then in the role of Chairman of its Executive Committee from 1985 until 1991. In 1992, Mr. Melamed became the founding Chairman of Globex. From 1993 to 2001, he served as Chairman and CEO of Sakura Dellsher, Inc., a former clearing firm of CME, and currently serves as Chairman and CEO of Melamed & Associates, a global consulting group. He is founder and a permanent advisor to the National Futures Association, and a member of the International Advisory Council of the CSRC in China. He serves on the board of overseers of the Becker Friedman Institute of the University of Chicago and on the advisory board of Vernon & Park Capital L.P. Melamed represents CME Group Foundation on the LEAP Innovations board of directors. He is also a published author of a number of books pertaining to markets and the history of CME Group.

MODERATOR

DAVID EISENBUD

Director, Mathematical Sciences Research Institute (MSRI)



David Eisenbud served as Director of MSRI from 1997 to 2007, and began a new term in 2013.

He received his Ph.D. in mathematics in 1970 at the University of Chicago under Saunders MacLane and Chris Robson, and was on the faculty at Brandeis University before coming to Berkeley, where he became Professor of Mathematics in 1997. He served from 2009 to 2011 as Director for Mathematics and the Physical Sciences at the Simons Foundation, and is currently on the board of

directors of the Foundation. He has been a visiting professor at Harvard, Bonn, and Paris. Eisenbud's mathematical interests range widely over commutative and non-commutative algebra, algebraic geometry, topology and computer methods.

Eisenbud is Chair of the editorial board of the "Algebra and Number Theory" journal, which he helped found in 2006, and serves on the board of the "Journal of Software for Algebra and Geometry," as well as Springer-Verlag's book series *Algorithms and Computation in Mathematics*.

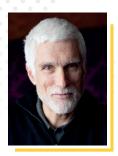
Eisenbud was President of the American Mathematical Society from 2003 to 2005. He is a Director of Math for America, a foundation devoted to improving mathematics teaching. He has been a member of the board of Mathematical Sciences and their Applications of the National Research Council, and is a member of the U.S. National Committee of the International Mathematical Union. In 2006, Eisenbud was elected a Fellow of the American Academy of Arts and Sciences.

$$\frac{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}}{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}} = E\{\sum_{I} D_{I}^{j} \mid p \& \sum_{\ell} s_{\ell} = b_{\circ}\}. \quad 0 = G_{j} - s_{j}' \frac{1}{\partial s_{I}'} \frac{1}{\partial s_{I}'} G_{j}^{*} \delta_{I}^{j} d\mu_{I} + [p-c_{j}]s_{j}' \sum_{j} \frac{1}{\partial s_{J}'} G_{j}^{*} \delta_{I}^{j} \delta_{I}^{j} d\mu_{I} + [p-c_{j}]s_{J}' \sum_{j} \frac{1}{\partial s_{J}'} G_{j}^{*} \delta_{I}^{j} d\mu_{I} + [p-c_{j}]s_{J}' \sum_{j} \frac{1}{\partial s_{J}'} G_{j}^{*} \delta_{I}^{j} \delta$$

PANELIST

DREW FUDENBERG

Professor of Economics, Massachusetts Institute of Technology



Drew Fudenberg is the Paul A. Samuelson Professor of Economics at MIT. He received an A.B. in applied mathematics from Harvard College in 1978, and a Ph.D. in economics from MIT in 1981.

He is a Fellow of the Econometric Society, and will be its President in 2017. He is a member of both the National Academy of Sciences and the American Academy of Arts and Sciences, and has received fellowships from the Guggenheim Foundation and the Alfred P. Sloan Foundation.

He is a past editor of "Econometrica" and a co-founder of the open access journal, "Theoretical Economics."

Fudenberg's work on game theory ranges from foundational work on learning and equilibrium to the analysis of repeated games and reputation effects to the study of particular games, competition between firms and other topics in theoretical industrial organization. More recently he has worked on topics in behavioral economics and decision theory such as self-control and stochastic choice. He is the author of four books: *Dynamic Models of Oligopoly* (1986) with Jean Tirole, *Game Theory* (1991) with Jean Tirole, *The Theory of Learning in Games* (1998) with David K. Levine, and *A Long-Run Collaboration on Long-Run Games* (2008) with David K. Levine.

PANELIST

SRIHARI GOVINDAN

Professor, Department of Economics, University of Rochester



Srihari Govindan is a Professor of Economics at the University of Rochester. He obtained his PhD in Economics from Stony Brook. He is a mathematical game theorist whose research interests are in the foundations of game theory. His current research focuses on axiomatic equilibrium selection in finite games and repeated games and the issue of existence of equilibria in games with non-compact spaces.

$$\frac{ \angle_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}}{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}} = E\{\sum_{I} D_{I}^{j} \mid p \& \sum_{\ell} s_{\ell} = b_{\circ}\} \cdot \bigcup_{-\left(s_{j} \sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I} + [p-c_{j}]s_{j}^{\prime} \sum_{j} \partial s_{j}^{\prime} \right)} G_{j}^{j}$$

PANELIST

BENGT HOLMSTRÖM

Nobel Laureate in Economic Sciences (2016), Paul A. Samuelson Professor of Economics, Massachusetts Institute of Technology



Bengt Holmström is the Paul A. Samuelson Professor of Economics at Massachusetts Institute of Technology, where he was head of the Economics Department from 2003-2006. He holds a joint appointment with MIT's Sloan School of Management. He is an elected fellow of the American Academy of Arts and Sciences, the Econometric Society and the American Finance Association, and an elected foreign member of the Royal Swedish Academy of Sciences and the Finnish Academy of Sciences and Letters. He is a research associate

of the National Bureau of Economic Research (corporate finance). In 2011 he served as President of the Econometric Society.

He received his doctoral degree from Stanford University in 1978. Before joining MIT in 1994, he was the Edwin J. Beinecke Professor of Management at Yale University's School of Management (1983-94) and associate professor at the Kellogg Graduate School of Management at Northwestern University (1979-82).

Holmström is a microeconomic theorist, best known for his research on the theory of contracting and incentives especially as applied to the theory of the firm, to corporate governance and to liquidity problems in financial crises.

He holds honorary doctorate degrees from the University of Vaasa, Finland; Stockholm School of Economics, Sweden and the Hanken School of Economics, Finland. He was awarded the Banque de France-TSE Senior Prize in Monetary Economics and Finance in 2012, the Stephen A. Ross Prize in Financial Economics and the Chicago Mercantile Exchange – MSRI Prize for Innovative Quantitative Applications in 2013, the Distinguished CES Fellow award from CESifo, Munich, and the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel in 2016.

PANELIST

PAUL MILGROM

Shirley R. and Leonard W. Ely Jr. Professor of Humanities and Sciences, Economics Department, Stanford University

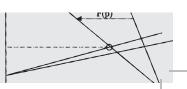


Paul Milgrom is the Shirley R. and Leonard W. Ely Professor of Humanities and Sciences in the Department of Economics at Stanford University, a member of the National Academy of Sciences, a Fellow of the American Academy of Arts and Sciences and winner of the 2008 Nemmers Prize and the 2013 BBVA Foundation Frontiers of Knowledge prize. According to Google Scholar, his research works have more than 75,000 citations, covering multiple fields in economics. A leader in radio spectrum policy and auction theory and

applications, Milgrom co-invented auction formats used for selling spectrum licenses in North America, Europe, Asia and Australia.

In 2009 Milgrom co-founded Auctionomics, a high stakes auction consulting and software firm that has helped governments worldwide design and implement complex auctions. It has also prepared companies across the globe to bid in complex auctions. Milgrom and Auctionomics recently led the design of the (currently running) U.S. Incentive Auction, which will buy TV broadcast licenses and sell wireless broadband licenses.

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PANELIST

ROGER MYERSON

Nobel Laureate in Economic Sciences (2007), Glen A. Lloyd Distinguished Service Professor of Economics, University of Chicago



Roger Myerson studied applied mathematics at Harvard University, earning his bachelor's degree in 1973 and his Ph.D. in 1976. From 1976 to 2001, he was Professor of Managerial Economics and Decision Sciences at Northwestern University, in the Kellogg School of Management. Since 2001, he has been at the University of Chicago where he is the Glen A. Lloyd Distinguished Service Professor in Economics. He is a fellow of the American Academy of Arts and Sciences and the National Academy of Sciences.

He is author of two books, *Game Theory* (1991) and *Probability Models for Economic Decisions* (2005), and many professional articles on game theory, information economics and political economics. In particular, he has written about bargaining problems with incomplete information, refinements of Nash's equilibrium concept, optimal auction design, incentive constraints in economic systems, and game-theoretic models of politics.

In 2007, he shared the Nobel Memorial Prize in Economic Sciences, in recognition of his contributions to mechanism design theory, which analyzes rules for coordinating economic agents efficiently when they have different information and difficulty trusting each other.

PANELIST

ALVIN ROTH

Nobel Laureate in Economic Sciences (2012), Craig and Susan McCaw Professor of Economics, Department of Economics, Stanford University



Al Roth is the Craig and Susan McCaw Professor of Economics at Stanford University, and the George Gund Professor Emeritus of Economics and Business Administration at Harvard. He shared the 2012 Nobel memorial prize in Economics.

His research interests are in game theory, experimental economics, and market design. He directed the redesign of the National Resident Matching Program, through which approximately twenty-five thousand doctors a year find

their first employment as residents at American hospitals. He has also helped in the reorganization of the market for more senior physicians as they pursue subspecialty training, and in other labor markets.

He helped design the high school matching system used in New York City to match approximately ninety thousand students to high schools each year. He also helped redesign the matching system used in Boston Public Schools, for students of all ages.

More recently he has helped design school choice systems in several other large American cities. He is one of the founders and designers of kidney exchange in the United States, which helps incompatible patient-donor pairs find life-saving compatible kidneys for transplantation.

$$\frac{ \sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}}{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}} = E\{\sum_{I} D_{I}^{j} \mid p \& \sum_{\ell} s_{\ell} = b_{\circ}\}. \quad 0 = G_{j} - s_{j}' \frac{1}{\partial s_{I}'} \frac{1}{\partial s_{I}'} G_{j}' \frac{1}{\partial s_{I}'} G_{j$$

LUNCHEON REMARKS



APPRECIATION OF THE LIFE AND WORKS OF ROBERT WILSON

PHILIP RENY

The Hugo F. Sonnenschein Distinguished Service Professor in Economics and the College University of Chicago

Professor Reny is the Hugo F. Sonnenschein Distinguished Service Professor of Economics, and is an economic theorist whose topics of interest include auction theory, information

aggregation, game theory and the theory of mechanism design. His current research focuses on the existence of Nash equilibrium in discontinuous games, methodologies for analyzing rational behavior in extensive form games with infinite actions and types, and optimal mechanism design with multi-dimensional private information.

Reny serves on the board of editors for the "American Economic Journal: Microeconomics," has served as the head editor of "Journal of Political Economy," and chaired the Department of Economics at the University of Chicago from 2006 to 2009. He became a member of the American Academy of Arts and Sciences in 2015, a Fellow of the Society for the Advancement of Economic Theory in 2012, a charter member of the Game Theory Society in 1999 and a fellow of the Econometric Society in 1996. He received his Ph.D. in Economics from Princeton University in 1988.

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Prize Committee Chair, Director, Mathematical Sciences Research Institute (MSRI)

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Nobel Laureate in Economic Sciences (2013) and David Rockefeller Distinguished Service Professor in Economics, Statistics and the College, University of Chicago and 2008 Recipient of the CME Group-MSRI Prize in Innovative Quantitative Applications

BENGT HOLMSTRÖM

Nobel Laureate in Economic Sciences (2016), Paul A. Samuelson Professor of Economics, Massachusetts Institute of Technology and 2013 Recipient of the CME Group-MSRI Prize in Innovative Quantitative Applications

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Charles and Lynn Zhang Professor of Economics, Columbia University; Theodore A. Wells '29 Professor of Economics Emeritus, Princeton University and 2014 Recipient of the CME Group-MSRI Prize in Innovative Quantitative Applications

MYRON SCHOLES

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Frederick Henry Prince Distinguished Service Professor of Economics, University of Chicago

JEAN TIROLE

Nobel Laureate in Economic Sciences (2014), Scientific Director of Industrial Economics Institute, Member of the Toulouse School of Economics and 2010 Recipient of the CME Group-MSRI Prize in Innovative Quantitative Applications

$$\frac{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}}{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}} = E\{\sum_{I} D_{I}^{j} \mid p \& \sum_{\ell} s_{\ell} = b_{\circ}\}.$$

$$\frac{\mathbf{O} = \mathbf{G}_{j} - s_{j}^{\prime}}{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I} + [p-c_{j}]s_{j}^{\prime} \sum_{I} \mathbf{O} s_{j}^{\prime}} \mathbf{G}_{j}^{j}$$

PAST CME GROUP-MSRI PRIZE RECIPIENTS

2015 | DOUGLAS DIAMOND

Fischer Black Visiting Professor of Financial Economics, MIT Sloan School of Management, 2015-2016 and Merton H. Miller Distinguished Service Professor of Finance, Booth School of Business, University of Chicago

2014 | JOSÉ SCHEINKMAN

Charles and Lynn Zhang Professor of Economics, Columbia University; Theodore A. Wells '29 Professor of Economics Emeritus, Princeton University

2013 | BENGT HOLMSTRÖM

Nobel Laureate in Economic Sciences (2016), Paul A. Samuelson Professor of Economics, Massachusetts Institute of Technology

2012 | ROBERT SHILLER

Nobel Laureate in Economic Sciences (2013), Sterling Professor of Economics, Yale University, and Professor of Finance and Fellow at the International Center for Finance, Yale School of Management

2011 | THOMAS SARGENT

Nobel Laureate in Economic Sciences (2011), William R. Berkley Professor of Economics and Business, Stern School of Business, New York University, and Senior Fellow, Hoover Institution

2010 | JEAN TIROLE

Nobel Laureate in Economic Sciences (2014), Scientific Director of Industrial Economics Institute, Member of the Toulouse School of Economics

2009 | SANFORD GROSSMAN

Chairman and President, Quantitative Financial Strategies, Inc.

2008 | LARS PETER HANSEN

Nobel Laureate in Economic Sciences (2013) and David Rockefeller Distinguished Service Professor in Economics, Statistics and the College, University of Chicago

2007 | DAVID KREPS

Adams Distinguished Professor of Management, Professor of Economics (by courtesy), School of Humanities and Sciences, and the Winnick Family Faculty Fellow for 2016-2017, Stanford University Graduate School of Business

2006 | STEPHEN ROSS

Franco Modigliani Professor of Financial Economics, MIT Sloan School of Management

$$\frac{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}}{\sum_{I} \int_{M(I,p)} f_{I}^{j} \delta_{I}^{j} d\mu_{I}} = E\{\sum_{I} D_{I}^{j} \mid p \& \sum_{\ell} s_{\ell} = b_{o}\}.$$

