CME's Average Pricing Algorithm

This document provides specifications for CME's **average pricing calculations**, often referred to as **APS** ("apes"), for its original implementation in the **Average Pricing System**.

Average-pricing calculations allow any number of transactions in a contract, on the same side of the market (only buys or only sells), to be offset and then replaced with one or more special transactions at an **average price**. The input transactions being averaged are considered an **average pricing group ("APS group")**.

Because many firm bookkeeping systems are limited in the price formats they support, the system has always been implemented with both the **true average price**, and the **rounded average price** and **residual**. The rounded average price conforms to normal price format rules for the contract, and the residual represents the monetary difference between the transactions at the true average price and at the rounded average price.

The process works as follows:

- The transactions to be averaged are selected and placed into the average-pricing group.

- The system calculates the true average price, the rounded average price, and the residual amount for the entire group.

- The total group quantity is then allocated out, via one or more allocations, either within the firm or to other firms.

- When each allocation is accepted (claimed), the system creates a pair of **APS transfer transactions** (an offset and an onset transaction) for each, at the rounded average price and with a portion of the total residual.

The offsets and the onsets are considered transfers, in exactly the same manner as transfers resulting from normal giveup or suballocation processing. In fact, giveups and average pricing may be considered as two sides of the same coin: the primary difference between normal giveups and average pricing, is that with normal giveups all of the incoming transactions are at the same price.

The algorithm has been designed to satisfy several criteria:

- The true average price must be calculated as the exact quantity-weighted average of the transactions in the APS group. It must not be distorted either by rounding or by monetary values that are fractional (less than a penny). In particular, if all of the transactions in the APS group are at the same price, then the true average price must be that same value.

- The residual amount must be the **exact** monetary difference between the total value of the transactions in the APS group at their original trade prices, and the total value of the replacement transaction at the rounded average price. If there is only one allocation, then the participants must neither gain nor lose monetary value as a result of the price averaging process.

- Because the incoming transactions may be replaced by more than one allocation, in some circumstances the monetary value of the allocations may not be precisely equal to the value of the incoming transactions, due to rounding. If this occurs, however, then the total value of the allocated residual amounts must be within pennies of the total residual value. In addition, the rounding mechanism is selected to follow the long-standing convention that "the executing firm keeps the pennies."

- The algorithm must work for contracts denominated in any currency, and regardless of what the normal precision is for amounts denominated in that currency.

Note also that the calculation does not depend on the total precision to which the true average price is carried. Normally the true average price is calculated as a double-precision floating-point value, and then rounded or truncated to some total number of digits for display purposes. In CME's implementation, the true average price is reported out to ten total decimal digits (ten digits to the right of the decimal point.)

A note on rounding and currency precision

The normal precision for most currencies – for example, USD, EUR, GBP, CAD, AUD, CHF – is two decimal places. In other words, "money amounts are carried out to the penny." The normal precision for JPY is zero decimal places – *ie,* to the nearest yen.

"Rounding normally" means when the value being rounded is precisely equidistant between two possible rounding points, you round **up** (**away from zero**).

Here's the algorithm, first in English and then algebraically:

For each trade in the average-price group:

> Decimalize the price as needed, yielding the **decimalized price**, with no loss of precision.
> Multiply the decimalized price by the trade quantity, yielding the **quantity-weighted price**.
>
> Multiply the decimalized price by the contract value factor.
> Round this result to the normal precision of the settlement currency of the product.
> Multiply this result by the trade quantity, yielding the **trade value**.

Take the sum across all such trades of:

> Quantity-weighted price, yielding the **total quantity-weighted price**
> Trade quantity, yielding the **total quantity**
> Trade value, yielding the **total trade value**.

Divide the total quantity-weighted price by the total quantity, yielding the **true average price**.

Round this result to the nearest tick applicable to this contract, rounding up if the group is composed of buys and down if the group is composed of sells, yielding the **rounded average price**.

> (for reports and screens, the rounded average may need to be in fractional format, but for these calculations, keep it in fully decimalized form)

Multiply the rounded average price by the contract value factor, and round this result to the normal precision of the settlement currency. Now multiply this result by the total quantity, yielding the **total value at the rounded average price**.

Subtract the total trade value from the total value at the rounded average price.
If averaging buys, this result is the **total residual value**. (the "residual")
If averaging sells, multiply this result by -1 to get the total residual value.

For each allocation:

> Divide the allocation quantity, by the total quantity.
> Multiply the total residual value by this ratio.
>
> Truncate this result ("round down") as needed to get a monetary value to the normal precision of the settlement currency, yielding the **residual for this allocation**. ("the executing firm keeps the pennies")

Note that normally the total residual is a positive value, meaning that the cash flow is from the executing firm to the carrying firm – ie, negative on the offset transfer transaction, and positive on the onset transfer transaction.

> However, there can be cases for CBOT products where the true average comes out exactly on a tick boundary, and the residual is a small but nonzero, **negative** value. In this case everything works as before except that the actual cashflow direction is reversed – from the carry firm to the executing firm.

The average-pricing algorithm expressed algebraically

Suppose there are **n** trades in the average-price group.
Let:

- **CVF** be the contract value factor
- **Qi** be the trade quantity for the i-th trade, and
- **Pi** be the fully-decimalized trade price for the i-th trade.
- **QWPi** be the quantity-weighted price for the i-th trade.
- **Vi** be the trade value for the i-th trade.

And define:

- **TWP** as the total quantity-weighted price
- **TTQ** as the total trade quantity
- **TTV** as the total trade value
- **TAP** as the true average price
- **RAP** as the rounded average price
- **TVRAP** as the total value as the rounded average price
- **RESID** as the residual

Then for each trade:

QWPi = Pi * Qi

Vi = round(Pi * CVF) * Qi

where the rounding is done **normally to the normal precision of the settlement currency**

And across the trades:

TWP = sum from i=1 to n: (QWPi)
TTQ = sum from i=1 to n: (Qi)
TTV = sum from i=1 to n: (Vi)

TAP = TWP / TTQ with no loss of precision

RAP = round(TAP)

Where the rounding is done to the **normal precision of trade prices** for this contract at this price level, and is rounded **up for buys** or **down for sells**

TVRAP = round(RAP * CVF) * TTQ

where the rounding is done **normally to the normal precision of the settlement currency**

RESID = TVRAP – TTV [* -1 if the group is composed of sells]

Now suppose we have m allocations of the total trade quantity. Let:

- **AQj** be the allocation quantity for the j-th allocation, and
- **RESIDj** be the value of the residual for the j-th allocation

Hence:

RESIDj = truncate(RESID * (AQj / TTQ))

Where the truncation results in rounding down to the normal precision of the settlement currency.

How "cabinet" trade prices affect the averaging process

For options with a fixed cabinet price (*ie,* CME options):

- The price that should go into APS is the price assigned as the "fixed cabinet price".

- There's no issue with the rounding. No trade can occur at less than the cabinet price, so if you are rounding down, you will never round down below the cabinet price.

For options with variable cabinets, ie, CBOT options, life gets more complicated. Here, you have to take the variable cabinet price – for example, 3.00 – which is actually a 3.00 **monetary value** – and divide by the contract value factor to get the trade price in price terms. Now, you can do the price averaging and determine the true average normally.

Special processing is needed for determining the rounded average. The variable cabinet prices are a range, for example from 1 to 16 for a particular contract, where, let's say, the first non-cabinet price is equivalent to 16.25 in money terms.

So the first thing is to see if the true average price is greater than or equal to the price at the first non-cabinet point. If it is, then you can determine the rounded average normally.

But if not, then you first convert the true average to its money value by multiplying by the contract value factor and rounding to the precision of the settlement currency.

Now look at the true average's money value. Is it greater than the highest cabinet price but less than the money value of the lowest non-cabinet tick? If so, then if you're rounding up, the rounded price is the lowest non-cabinet tick, and if you're rounding down, the rounded value is the highest cabinet price.

If the true average's money value is less than or equal to the highest cabinet price, then you can round normally to one of the allowable cabinet prices.

Keep in mind when doing the residual calculation that the value at a variable cabinet price, is the price.

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