CME Options Workshop

Week 5: Black-Scholes

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Let’s start with a poll

What is the single most important factor in option pricing?

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Let’s start with a poll

What is the single most important factor in option pricing?

A. Futures
B. Volatility
C. Interest rates

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Black-Scholes

Understand the inputs to the model and determine what you can forecast differently (and correctly) vs. the market.

View on interest rates or implied volatility or simply direction?

Sense of timing of an event?

How can this help you express your view?

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Black-Scholes model

- B-S equation comes from physics (heat transfer equation)
- Like all models, it makes some simplifying assumptions i.e. that interest rates and volatility are constant over the life of the option
- If this were the case, we know then that:
  - -Change in option value depends on the sensitivity to change in future (delta), sensitivity of delta to change in future (gamma) and sensitivity to the change in time (theta)

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Black-Scholes model

- Interest rate has two components:
  - -Price of the forward
  - -Discounted value of option value at any time in life
- Volatility has two components:
  - -The extent to which it affects future intrinsic value
  - -The effect it has on the Greeks in the model: delta, gamma, theta

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Black-Scholes

$$C = S\,N(d_1) - Ke^{-rt}\,N(d_2)$$

$S \cdot N(d_1)$ = option value expressed in terms of the futures price

$Ke^{-rt} \cdot N(d_2)$ = option value expressed in terms of the strike price

d_1 and d_2 are the z values from the table of standard normal values, and $N(d_1)$ and $N(d_2)$ are the corresponding probabilities

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Black-Scholes

$$C = S\,N(d_1) - Ke^{-rt}\,N(d_2)$$

C = call premium
S = price of underlying future
K = strike price
N = normal distribution operator
d_1 = expression 1
d_2 = expression 2

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Black-Scholes

$$d_1 = \left[\frac{\ln(S/K) + (r + .5\ \sigma^2)t}{\sigma\sqrt{t}} \right]$$

$N(d_1)$ = delta = change in option premium from a small change in the underlying future price; more on this later

$S \cdot N(d_1)$ = option value expressed in terms of the future price

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Black-Scholes

$$d_2 = d_1 - \sigma\sqrt{t} = \left[\frac{\ln(S/K) + (r - .5\,\sigma^2)t}{\sigma\sqrt{t}}\right]$$

$N(d_2)$ = probability that S is greater than K (i.e., option expires in the money)

$Ke^{-rt} \cdot N(d_2)$ = option value expressed in terms of the strike price

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Put-Call Parity

- The net cost of buying a synthetic option must equal the net cost of buying the “actual” option
- The net cost of buying a synthetic asset must equal the net cost of buying the “actual” asset
- The net cost of buying a synthetic futures contract must equal the net cost of the “actual” forward (or futures) contract
- These relationships reflect “The Law of One Price”

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Put-Call Parity

- The net cost of buying the asset using options must equal the net cost of buying the asset using a futures contract
- Call (K, t) – Put (K, t) = PV ($F_{0,t}$ – K)

  where Call (K, t) and Put (K, t) denote the premiums of options with strike price = K and time to expiration = t, and PV ($F_{0,t}$) is the present value of the forward price

  Positive = long position, Negative = short position
- **This is one of the most important relationships in options!**

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Put-Call Parity

- Call (K, t) – Put (K, t) = PV ($F_{0,t}$ – K)
- PV ($F_{0,t}$ – K) = PV($F_{0,t}$) − PV (K)
  - PV($F_{0,t}$) = S_t
  - PV (K) = Ke^{-rt}

So:

- Call (K, t) – Put (K, t) = S_t − Ke^{-rt} prior to expiration (t > 0)
- Call (K, t) – Put (K, t) = S_t − K at expiration (t = 0)

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Interaction time:

How do you form a view on the inputs to the model?

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Trading the inputs to the model

- What are the inputs to Black-Scholes?
- Future
- Strike
- Distribution
- Rates
- Time
- Volatility

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Trading the inputs to the model - direction

- Why do you need a view on the direction of the underlying?
- How do you come up with a view on the direction of the underlying?
- Fundamental
- Technical
- Event-driven
- Arbitrage / Mean-reversion

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Trading the inputs to the model - direction

Bring up your favorite technical tool

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Trading the inputs to the model – implied volatility

- How do you come up with a view on implied volatility?
- Serial correlation
- Mean reversion
- Supply and Demand
- Volatility forecasting – ARCH, GARCH, EWMA

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Importance of Volatility

- Implied vs. realized volatility
- Forecasted volatility
- Implied less realized volatility
- Intrinsic/Parity value + Time/volatility value
- Vega and gamma across the curve
- Skew
- Risk premia capture

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Importance of volatility

- If we buy a straddle and implied volatility goes immediately higher, all else equal, we would make money
- If, however, it takes several weeks for implied volatility to move higher, all else equal, we would lose money because of theta
- If volatility moves lower, but the underlying price moves away from current, we can make money
- Volatility and weather: serial correlation – what is your estimate for tomorrow's temperature?
- Volatility and weather: mean reversion – what if I told you the average temp this time of year?
- Volatility forecasting / term structure of volatility
- Implied volatility as a predictor of future volatility

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What about volatility?

Let’s bring back CVOL

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Skew and kurtosis

- Sticky strike skew
- Floating skew
- Curve resets
- Kurtosis aka ‘fat tails’
- Y = a + bX + cX^2: a is base vol, b is skew, c is kurtosis
- How can skew and kurtosis impact the ‘Greeks’?
- Volatility surface
- How does the shape of the volatility surface impact your trading strategy?
- Implied probabilities from butterflies

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What about volatility?

We can also use QuikStrike to look at the volatility surface

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Trading the inputs to the model – interest rates

- Why do you need a view?
- When you trade stocks or commodities, do you take into account rates?
- Do all investors need this view?
- Do all option traders need to have a view?

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What about interest rates?

Let’s look at the Treasury analytics at CME

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Trading the inputs to the model – time & catalysts

- Maybe the most important view of all to have
- Remember that options are time and path dependent
- How does one develop this view?
- Is this view something that is set in stone?

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What about events?

Let’s look at the economic calendar in CME tools

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Trading the inputs to the model – pulling together

- How do you incorporate this into a single view?
- Do you understand where spot and vol are now?
- Do you understand how moves in either affect each other?
- Do you understand where end-user demand is now?
- Do you understand how moves in spot and vol affect demand?
- Do you understand how time and catalysts affect the moves?
- Do rates matter all the time?

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Black-Scholes model short-cuts

- Black-Scholes makes all calculations assuming a normal distribution
- This may seem inconsistent with prices being lognormally distributed
- However, we can assume **returns** are normally distributed to get around this
- The main point is that the curve represents 100% of all occurrences
- We know that over any period, one standard deviation is equal to **$S*\sigma\sqrt{T}/365$**
- This is an important idea because then we can know what move is being priced in for any period by using $\sqrt{T}$ e.g. a 100 implied volatility is 6.3% per day

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Black-Scholes problems

- Clearly there are real problems: markets are not frictionless, volatility may change over the life of the option, prices do not follow a diffusion process etc.
- How do traders adjust to these short-comings?
- Volatility skew or volatility smile
- End-users in equities prefer the protective put and the covered call
- End-user in interest rates: the borrower or the lender get nervous
- In agriculture you have farmers vs. food companies
- In oil markets you have producers vs. refiners
- The end user tends to dominate

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Let’s end with a poll

What is more important for Black-Scholes?

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Let’s end with a poll

What is more important for Black-Scholes?

A. Knowing the formula
B. Knowing how the inputs affect the formula

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Questions?

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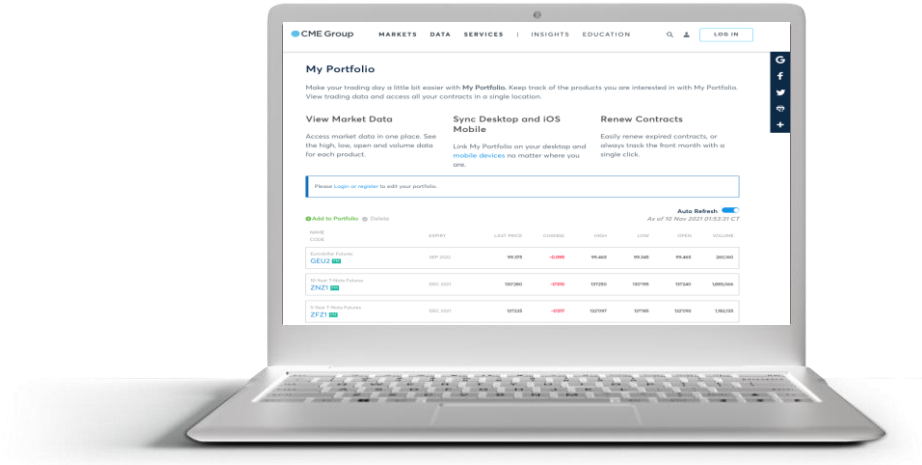

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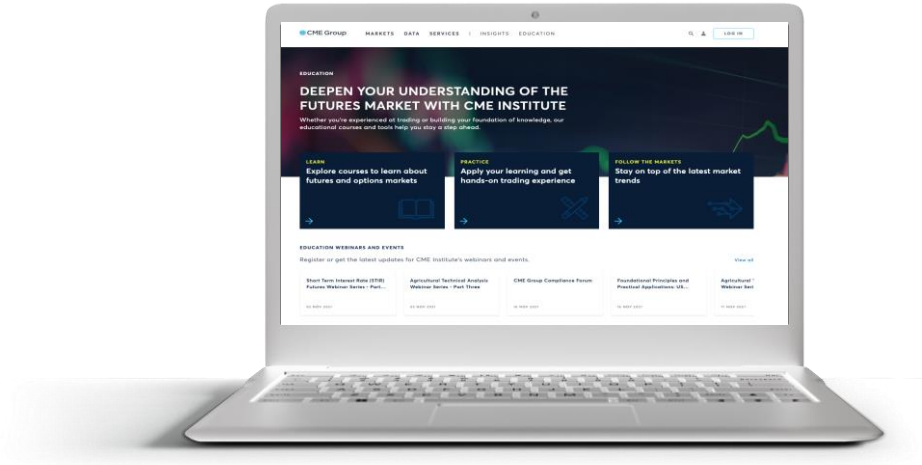

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