

# Empirical Duration

*Measuring the price sensitivity of U.S. Treasury Futures.*

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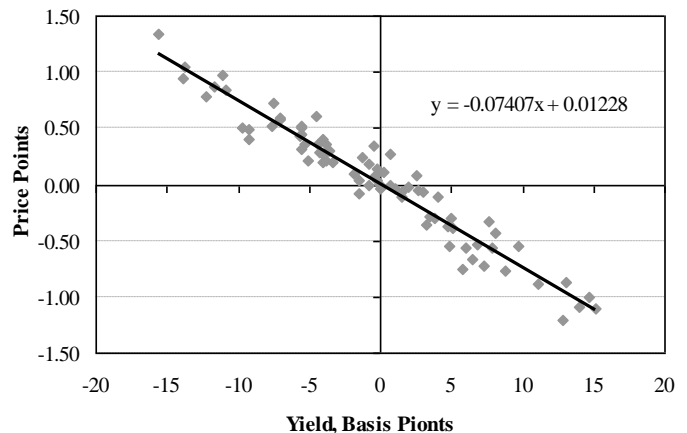
Treasury futures trade in price, but it is often useful to consider strategies in terms of yield changes. The dollar value of a one basis point change in yield for Treasury notes changes with yields. Treasury futures contracts reflect this same dynamic, as well as another: the note most prominently reflected in the contract may also change. Keeping track of the potential for duration drift in the contracts requires either direct observation, or some type of model to evaluate performance over a range of possible future scenarios. This paper details the method of direct observation employed at [cmegroup.com/duration](http://cmegroup.com/duration).

The two different approaches to measuring duration in instruments with embedded options, like Treasury futures, are analogous to looking forward or backward in time. Direct observation of an empirical history requires at least several weeks worth of prices, whereas forward looking models don't look to past prices. While model building can be subjective, there is no arguing with historical observation: it happened. One question that needs to be addressed after careful measurement of past interest rate sensitivity is whether or not the past is a relevant guide to the future.

The first step to measuring empirical duration of Treasury futures is to construct a time series of price changes in the contract and yield changes of the note most prominently reflected in the contract (the note traders often refer to as the "cheapest" note to deliver). Duration is literally the change in price of an instrument for a given change in yield, and this sensitivity can be measured with simple regression.

Figure 1 illustrates the price change of a 10-year Treasury futures contract against the yield change of the note most prominently reflected in the contract during the 60 prior trading days illustrates. Determining a historical measurement period is important, and as we will see the measurements are sometimes sensitive to this selection. For practical purposes there is seldom more than three months when a contract is actively traded, so this is the effective maximum length of any observation period. Measuring the price performance of a contract prior to this period of active trading is likely to lead to spurious results.

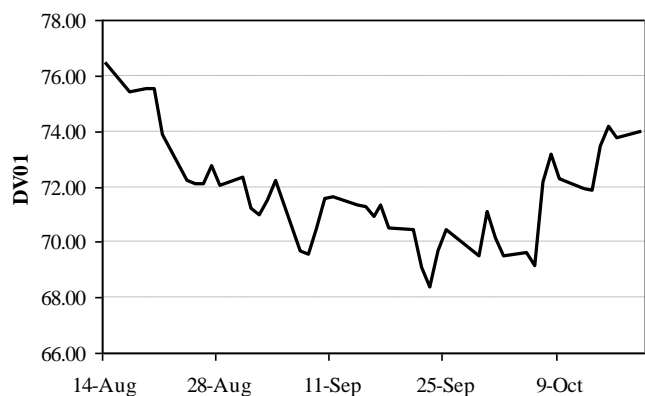
*Figure 1: 10-Year Futures price vs. cheapest Treasury note*



The slope of this regression line is what we are interested in, since it calculates the tradeoff between price and yield. The slope has to be scaled appropriately based on the face value of the contract. In the case of 10-year Treasury futures the slope of -0.07407 translates to the dollar value of a 1 basis point change in yield of \$74.07.

Further examination of this value gives us confidence that the results are relatively stable, and not heavily dependent on the historical observation window. Put another way, if we choose a shorter window than the past three-months, the value doesn't change significantly, as illustrated in Figure 2.

*Figure 2: Rolling 1-month history from Figure 1*



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Figure 2 illustrates the slope of the regression line from Figure 1 using a rolling 1-month window. The value has been scaled to reflect the per-contract dollar value and the negative sign is dropped. The values in Figure 2 fluctuate from a high of about \$76 to a low of \$68, reflecting a 12% swing. While the relatively short 1-month observation window is certainly introducing some volatility, some of the fluctuations in sensitivity deserve to be there.

The price sensitivity of the Treasury note in question, 4.875 Aug '16, is about \$71. The lowest values in Figure 2 hover around this mark, and briefly dip below it, but for most of September the risk of the futures contract mirrors the risk of the Aug '16 note. The empirical results indicate that the performance of the contract is in fact anchored by the underlying market. Values higher than \$71 reflect some risk of a change in the note most closely reflected in the futures contract, in this case to a longer-maturity note with a higher duration and price sensitivity above \$71. Rather than jump from reflecting the sensitivity of one note or another, the futures contract risk slides to reflect the characteristics of both issues, although one note usually heavily influences the characteristics of the contract.

Interest rate sensitivity of Treasury futures is especially important during calendar spread trading, where open interest moves from one expiration to the next. Often, there are different Treasury notes reflected in different futures expirations. For example, Aug '16 is prominently reflected in the December expiration, but September '16 is reflected in the March expiration. Different notes imply different sensitivities, and the difference between the two contracts is normally referred to as the "tail" by spread traders. The tail is automatically calculated by the web tool as the simple percentage difference between the front and deferred months, December and March in our example.

### **Price to Yield Calculator**

Treasury futures trade in terms of price, but common strategies are best understood in terms of yield: what strike option do I need if yields move 25 basis points? In order to answer these questions it's necessary to know the following inputs (and their source listed in parentheses):

1. Futures contract price (prior day's settlement)
2. the interest rate sensitivity (empirical duration)
3. proposed yield scenario (user input)

The web calculator automatically populates with the prior day's settlement price and the empirical duration for each Treasury futures contract outstanding. Users enter in a basis point change, assuming parallel shifts, and the output is a hypothetical futures price.

For example, suppose 10-year yields are 3.50%. What strike put is needed to protect against yields rising to 3.75% or higher? Suppose the current price of the contract is 110-00 and the empirical duration of the December 2009 10-year Treasury future is measured to be \$78.125 using the web tool, which is equivalent to 2.5 thirty-seconds per basis point in yield. In this case we estimate that 25 basis points higher in yield would lead to a contract price that is 62.5 thirty-seconds lower in price. The dollar price in this case has dropped almost two full points to 108-1+, which suggests that the 108 strike put would protect against parallel moves in yields to 3.75% or above.

Limitations of this approach are worth noting. First, the tradeoff between price and yield is not linear, although we are assuming it is based on the duration calculation. The implication is that the calculator will produce accurate results for reasonably small changes in yield, but is less accurate for larger moves. Additionally, the question of convexity in an instrument with embedded options is a complex one, and is not fully captured by empirical duration. It is also often the case that the maturity of the note most prominently reflected in the contract is not the on-the-run note. Strategies suggesting on-the-run yields change by a certain amount imply parallel moves in the yield curve with this type of analysis.

Aside from the question of convexity and how the yield curve moves, Treasury futures are forward contracts that are sensitive to changes in financing rates. Unexpected changes in Fed Funds or LIBOR may be reflected in general collateral and special repo rates used to price the contracts. Discrete influences like this are likely to get lost in a historical measurement. Additionally, as a physically delivered contract, this analysis would not capture unexpected changes in the expected size of deliveries.

While most deliveries in the long history of Treasury futures have been relatively small, there have been times when unexpectedly large deliveries adjusted the market's expectation of the duration of the contracts. A companion tool at [cmegroup.com/tools](http://cmegroup.com/tools) tracking open interest changes over the course of calendar spread trading might be used in conjunction with the empirical duration tool to monitor this particular risk.

While empirical duration is an incomplete description of the behavior of Treasury futures, the strongest argument in favor of its use is that it captures actual historical price performance. Put another way, it is often less relevant to explain how an instrument should be performing than measuring how it has actually performed.

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