Forecasting Daily Volatility Using Range-based Data

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Abstract

Users of agricultural markets frequently need to establish accurate representations of expected future volatility. The fact that range-based volatility estimators are highly efficient has been acknowledged in the literature. However, it is not clear whether using range-based data leads to better risk management decisions. This paper compares the performance of GARCH models, range-based GARCH models, and log-range based ARMA models in terms of their forecasting abilities. The realized volatility will be used as the forecasting evaluation criteria. The conclusion helps establish an efficient forecasting framework for volatility models.

Keywords: range-based estimator, log range, GARCH models, ARMA models, forecast

1. Introduction

Users of agricultural markets always need to establish accurate representations of expected future volatility. For example, future volatility is the main ingredient is calculating expected daily optimal hedge ratios. The application of misspecified future volatility has the potential to induce inappropriate or even serious assessment of asset risk and portfolio selection. Thus, not surprisingly, seeking good volatility forecasts of agricultural market volatility has drawn increased attention from financial academics and practitioners.

On the one hand, the existence of volatility clustering at different frequencies has been extensively documented in the finance literature. This high degree of volatility persistence suggests that financial market volatility is predictable. On the other hand, forecasting the future level of volatility is challenging for several reasons. For example, volatility is not directly observable; therefore the choice of evaluation metric for forecasting performance is uncertain. Establishing an appropriate framework for volatility forecasting is an important theme for financial academics and is of great relevance to practitioners.

Numerous papers have employed ARCH (GARCH) models for forecasting. The ARCH family of models is specifically designed to model volatility clustering effects, and its use in forecasting is quite common. However, the forecasting performance of the ARCH family is rather controversial. Contrary to studies seeking new model specifications, Andersen and Bollerslev (1998) argue that ARCH models provide good out-of-sample forecasts. However, ex-post volatility measures may not provide correct appraisal of performances of volatility models. Thus, establishing more efficient volatility measures is useful to evaluate the forecasting ability of existing time series models. They reveal that the coefficient of multiple determination, R^2, is low when the daily squared returns are used as a measure of ex-post volatility. They show that realized volatility, the sum of intraday squared returns, is a much more efficient volatility proxy.

The fact that range-based volatility estimators are highly efficient has been acknowledged by many authors (For example, Parkinson (1980), Garman and Klass (1980), Beckers (1983)). Their findings raise the question as to whether the forecasting ability of ARCH models can be improved with range-based data. However, these earlier works only focus on constructing efficient volatility estimators and little attention is paid to the application of these estimators. It was not until Alizadeh, Brandt and Diebold (2002) that the usefulness of a simple volatility proxy, the log range, was formally established and applied to time series models. They clarify that the log range, defined as the log of the difference between the high and low log prices during the day, is nearly Gaussian, robust to microstructure noise and much less noisy than alternative volatility measures such as log absolute or squared returns. Compared with earlier studies, their work fully exploits the distributional properties of the log range estimators and thus provides a theoretical underpinning for using the Gaussian ARMA class of models.

Previous findings related to range-based volatility estimator are essentially statistical. It is not clear whether using range-based data leads to better investment management decisions (e.g. , more accurate optimal hedge ratios). Furthermore, end-users of agricultural markets may not see additional benefits from range-based models when compared to extant simple volatility forecast framework, e.g., the Riskmetrics™ model.

Information costs may make range-based GARCH models undesirable. If an ARMA model of the log range has competitive forecasting ability, it will hold promise for practical applications of range estimators in agricultural asset pricing and risk management applications.

Motivated both by the appeal of range-based models and by the practical need to check their forecasting ability in agricultural commodity futures market, this study will investigate whether ARCH models extended with the range data provide better out of sample forecasts of daily volatility and whether a simple ARMA model of the log range has competitive forecasting ability. More importantly, this study will check the forecasting performance under different criteria using realized volatility.

The remainder of this paper is organized as follows. Section 2 presents the time series models used in this study. Section 3 summarizes the data and the in sample fit of the models. Section 4 presents the forecasting results and compare results under different criteria. Section 5 concludes.

2. Time Series Models

2.1 GARCH Models

There are two major categories of time-varying volatility models, the ARCH family and the stochastic volatility (SV) family. The autoregressive conditional heteroskedasticity (ARCH) model was introduced by Engle (1982). Compared with previous econometric models, ARCH processes are specifically designed to model and forecast conditional variances. This property of the ARCH model makes it appealing for modeling the volatility of economic time series. Bollerslev (1986) proposed an extension of the conditional variance function and introduced the generalized ARCH (GARCH) model. For many applications, the GARCH (1,1) model has been proved to be a parsimonious representation that fits data well. The representation of the GARCH (1,1) is

$$y_t = c + \varepsilon_t \tag{1}$$

$$\varepsilon_t = \sigma_t \eta_t$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

where η_t is a mean-zero, unit-variance, i.i.d. random variable.

Engle and Kraft (1983) were the first to consider the effect of ARCH (GARCH) on forecasting. Akgiray (1989) was the first to apply the GARCH model to forecast volatility. After that, numerous papers have employed this method. However, the forecasting performance of the ARCH family is disappointing in many studies. Different conclusions have been drawn for different sample periods and different speculative markets. Andersen and Bollerslev (1998) provide a few insights into these results. They find that ARCH family performs better if the ex-post volatility is estimated by the sum of intraday squared returns.

2.2 GARCH Models Extended with Additional Information

Andersen and Bollerslev (1998) demonstrate that the daily squared return is a very noisy estimator. If the previous trading day is quite volatile, but the closing price happens to be the same as the opening price, the lagged daily squared return would be zero. Thus by extending the daily GARCH model with information related to the real volatility dynamics, the new model will provide a reasonable explanation that the previous day was volatile. The conditional variance equation may be extended to allow for the inclusion of additional regressors. For example, Bessembinder and Seguin (1993) include daily volume. Laux and Ng (1993) include the number of price changes. Taylor and Xu (1997) and Martens (2002) use intraday returns. Martens (2001) use daily range. The specification of the variance equation for the extended GARCH models is,

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \zeta I_{t-1} \tag{2}$$

where I_t presents any trade related variables such as the traded volume, the sum of squared intraday returns or the daily range.

2.3 Simple Regression Model

The Stochastic Volatility (SV) models are more flexible than the ARCH family in the sense that volatility is driven by a noise term which may or may not be related to the returns process (Poon and Granger, 2001). However, since SV models involve an unobservable, stochastic variance process, this precludes closed-form likelihood functions; in turn estimation of SV models is quite difficult. The idea of using the Simple Regression (SR) model instead of the SV model to forecast volatility comes from Poon and Granger (2001). They suggest that

> "One way to avoid this [SV] estimation problem is to abandon the structure of the mean and express the volatility simply as a function of its past values. This is known as the Simple Regression (SR) method. The SR method is principally autoregressive. If past volatility errors are included, one gets the ARMA model for volatility."[1]

It would be interesting to explore whether alternative volatility proxies, such as the log range and squared intraday returns, fit the class of SR models. The logic is that if an estimator is highly efficient, it is possible to extract valuable information about the future

[1] Page 8.

value of volatility by just using simple technique. Given the findings described by Alizadeh, Brandt and Diebold (2002), it is natural to assume that the log-range process falls within the Gaussian ARMA models. If true, this will greatly reduce the computational costs. Standard forecasting techniques may be applied to generate predictions of future log range. Through simple transformations, the forecasts of volatility can be obtained. Specifically, the SR model for the range data is,

$$R_t = \alpha_1 R_{t-1} + \alpha_2 R_{t-2} + \ldots + v_t \tag{3}$$

where $v_t \sim iid(0,1)$; R_t denotes the log range.

3. Data and In-Sample Fit

3.1 Data Description

The data set consists of Chicago Board of Trade (CBOT) soybean futures intraday transaction prices and daily prices. The daily data were obtained from the CRB/Bridge Futures Database. The sample consists of daily soybean futures high/low/closing prices from January 2, 1985 to July 31, 2001. The intraday data are time and sales transaction prices, which were obtained from the Futures Industry Institute. The full sample covers the period January 2, 1990 to July 31, 2001. The first 1,264 trading days (January 2, 1985—December 29, 1989) are used to estimate the parameters of the various models. The next 2,909 trading days (for which intraday data are available) are used to test the out-of-sample forecasting performance.

In calculating the returns series, the nearby contracts are used to construct the continuous returns series. However, returns are calculated from the second nearby contract when the nearby contract is in the delivery month. This switch guarantees that returns are nearly always calculated from the prices of the contract that has the highest trading volume.

Figure 1 plots the prices of the futures data. Figure 2 plots the returns series actually used in this study, which is $100 \times \ln(P_t / P_{t-1})$ of the futures data. Table 1 reports summary statistics for daily returns. Soybean futures returns conform to several stylized facts which have been extensively documented for financial variables. The distribution of the returns is almost symmetric and has fat tails and a substantial peak at zero. Excess kurtosis of the series indicates that the distribution of daily returns is far from Gaussian. The autocorrelations of returns are close to zero. The Q-statistics are smaller than the critical values at 5% level. In contrast, the squared returns are significantly autocorrelated. Figure 2 reflects another stylized fact, the clustering effect. Variances of returns change over time and large (small) changes tend to be followed by large (small) changes.

In this study, the daily range is defined as,

$$Range_t = Max(h_t, c_{t-1}) - Min(l_t, c_{t-1}) \qquad (4)$$

where h_t and l_t denote highest and lowest prices on day t respectively and c_{t-1} represents the closing price on day $t-1$. Since the current soybean daily data only cover the full floor trading from 9:30 a.m. to 1:15 p.m., equation (4) captures information about the overnight market activity. While some of the markets previously studied in the literature do have trading limits in place, such as the S&P 500, they are much less frequently invoked than in physical commodity markets. When the equilibrium price moves beyond the trading limits, trading ceases. Since no trades are recorded during these moves, equation (4) provides a reasonable range proxy for limit move days. Following Alizadeh, Brandt and Diebold (2002), the daily log range is defined as,

$$R_t = \log[\log(Max(h_t, c_{t-1})) - \log(Min(l_t, c_{t-1}))] \quad (5)$$

The volatility literature primarily uses absolute or squared returns as volatility proxies. To justify the superior efficiency of the log range, Table 2 presents descriptive statistics for log absolute returns and the log range. Firstly, the log range is preferable in terms of its smaller standard deviation. Secondly, the skewness and kurtosis of the log range are 0.2405 and 2.9312, respectively. These values are closer to the corresponding values of 0 and 3 for a normal random variable compared with those for log absolute returns. This conclusion is confirmed by checking the Jarque-Bera statistic. It is more obvious by looking at Figure 3, which shows the quantile-quantile (Q-Q) plot. The Q-Q plot for the log range falls nearly on a straight line and indicates that the log range has a distribution close to Normal. In contrast, the Q-Q plot of the log absolute returns curves downward at the left end and upward at the right. Finally, the log range proxy is superior in terms of its time series dynamics. The large and slowly decaying autocorrelations of the log range clearly manifest strong volatility persistence. The erratic fluctuation of log absolute returns masks the volatility persistence.

3.2 In-Sample Fit

3.2.1 Estimation of GARCH Models

Maximum likelihood estimation of the GARCH model is easy to implement once the density function of ε_t is specified. If the residuals are not conditionally normally distributed, quasi-maximum likelihood (QML) estimator will still be consistent provided that the mean and variance functions are correctly specified.

The seasonal effects of price volatility are widely documented in many surveys. In time series modeling, one can take care of seasonality first and fit a model with the deseasonalized data. Or a model can be estimated for seasonally unadjusted data by adding a seasonal component in the model. This study follows the second approach. Roberts (2001) models the seasonal effects in volatility by including a Fourier expansion for the intercept of the GARCH volatility equation. The specification of the GARCH model is thus of the form,[2]

$$100 \times \ln(P_t / P_{t-1}) = \mu + \varepsilon_t \tag{6}$$

$$\varepsilon_t = \sigma_t \eta_t$$

$$\sigma_t^2 = \omega_t + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

$$\omega_t = \kappa + \sum_{m=1}^{M} \phi_m \sin(2m\pi\tau) + \psi_m \cos(2m\pi\tau) \qquad 0 \le \tau \le 1$$

where τ denotes the time of year of the observation.

Estimation results (based on 1985-1989 daily data) for GARCH (1,1) models are given in Table 3. All of the specifications capture well the autocorrelation in the volatility of returns. For the GARCH (1,1) model without seasonality, the estimates of parameters α and β are highly significant. The persistence in volatility is quite large, with $\alpha + \beta$ larger than 0.98. The Ljung-Box portmanteau test statistic for up to tenth order serial correlation in the standardized residuals η_t takes the value Q (10) = 8.0294, which is not significant for the χ_{10}^2 distribution. However, the Q-tests suggest that there exists serial dependence in the residuals squares at lag 5 and lag 10. Furthermore, the

[2] GARCH (1,1) model is considered to be a parsimonious representation, since results not reported here show that higher orders have nothing extra to offer.

unconditional sample kurtosis for the residuals is 4.0142, which exceeds the normal value of three. And the residuals continue to display asymmetry.

The second set of results in Table 3 include a first order seasonal expansion. Only one of the two seasonal parameters is significant at 5% level. However, the LR test statistic equals 9.04, which is significant at 2.5% level in the corresponding asymptotic χ_2^2 distribution. The addition of two parameters is also preferred from an AIC (Akaike information criteria) perspective. Moreover, the inclusion of a Fourier series reduces the sample kurtosis in residuals.

For the second order seasonality, only two seasonal parameters are significant at 5% level although the LR test and AIC prefer the inclusion of two additional parameters. Including a third order seasonality is rejected not only from a LR test perspective but also from a t-test perspective.

Note, the estimated value for α decreases as more parameters are added into the variance equation. This indicates that the reliance of the conditional volatility on the previous date is reduced. From the above results, a first order seasonality model represents a reasonable tradeoff between the need of model fit and the need of parsimony. Also the GARCH (1,1) model with no seasonality is used as the reference basis for forecast analysis.

3.2.2 GARCH Models Extended with Daily Range

In this section, the daily soybean data are fitted to the GARCH models extended with daily range. The intercept of the variance equation is defined as,

$$\omega_t = \kappa + \zeta I_{t-1} + \sum_{m=1}^{M} \phi_m \sin(2m\pi\tau) + \psi_m \cos(2m\pi\tau) \tag{7}$$

where $I_t = Max(h_t, c_{t-1}) - Min(l_t, c_{t-1})$.

Table 4 presents the estimation results. First, the log-likelihood values of three GARCH-I models are greater than those of the corresponding GARCH models in section 4.2.1. This result suggests that range data improve in sample model fitting and reflects the greater precision of the range as a volatility proxy. Second, GARCH-I models are also desirable from the AIC and SC (Schwartz criteria) perspectives since both values of AIC and SC fall as the daily range is added in. Third, the estimates of α become not significant at 1% level or even at 5% level for second order and third order seasonality. In contrast, the estimates of range parameter ζ are highly significant. This result is foreseeable since ε_{t-1}^2 and I_{t-1} are competing factors to present last period's variance and the inclusion of range data reduces the proportions of ε_{t-1}^2 in accounting for last period's volatility. This result also confirms the fact that daily range is a relatively less noisy volatility proxy than daily squared returns.

Omitting range, the estimation results for the GARCH-I models are quite similar to the results for the GARCH models in terms of seasonality. AIC, SC and LR test all suggest that GARCH-I (1,1) with third order seasonality is readily rejected. The addition of one order of seasonality performs better than the second order seasonality model in terms of LR test. This conclusion is confirmed by the estimates of second order seasonality model. ϕ_2 is significant at 5% level and ψ_2 is not significant judged by the standard errors. Additionally, the values of AIC are close for both models. Finally, the skewness and kurtosis coefficients of the standardized residuals for three models do not provide much information about model selection. The p-values for Q-test are also close and tell a similar story for four models. All in all, the first order seasonality model works best for GARCH-I models. However, in order to avoid the possible over-fitting problem in forecasting, GARCH-I model without seasonality is also included as one of the forecasting frameworks.

4. Out-of-Sample Daily Volatility Forecasts

4.1 Forecast Evaluation Criteria

It is difficult to compare forecasting performance of competing models since there is a variety of evaluation criteria used in the literature. Statistical analysis is one of the evaluation measures frequently used. Poon and Granger (2001) suggest that utility-based economic criteria are costly to apply and statistical analysis provides a practical way for forecast evaluation. West and Cho (1995) consider alternative statistical measures. Basically, statistical measures evaluate the difference between forecasts at time t and realized values at time $t+k$. However, asset price volatility is not directly observable and measuring the realized values of volatility is challenging. Much effort has been devoted to extracting volatility from other observable market activities. The daily squared return has been widely used in the literature as ex-post volatility. However, Andersen and Bollerslev (1998) show that it is a very noisy volatility estimator and does not provide reliable inferences regarding the underlying latent volatility in daily samples. They introduce a new volatility measure, termed realized volatility. Realized volatility estimates volatility by summing squared intraday returns. Volatility estimates so constructed are close to the underlying integrated volatility. Thus, the volatility of a price process can be treated as an observable process. In this study, realized volatility is calculated based on 5-minute return series.

For the performance evaluation, two forecast evaluation criteria, Root Mean Square Error (RMSE) and Mean Absolute Error (MAE), are defined by,

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\sigma}_t^2 - \sigma_{rv,t}^2)^2} \tag{8}$$

where T denotes the forecast horizon. $\hat{\sigma}_t^2$ denotes one step ahead daily forecast and $\sigma_{rv,t}^2$ denotes realized volatility.

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|\ \hat{\sigma}_t^2 - \sigma_{rv,t}^2\ | \tag{9}$$

In order to account for the heteroskedasticity, two alternative measures, the heteroskedasticity adjusted root mean squared error (HRMSE) and mean absolute error (HMAE) are included. These two measures are computed as follows,

$$HRMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(1 - \frac{\sigma_{rv,t}^2}{\hat{\sigma}_t^2})^2} \tag{10}$$

$$HMAE = \frac{1}{T}\sum_{t=1}^{T}|\ 1 - \frac{\sigma_{rv,t}^2}{\hat{\sigma}_t^2}\ | \tag{11}$$

The second metric used to evaluate daily volatility forecasts is the regression-based method. The coefficient of determination (R^2) of the regression of realized volatility on forecasted volatility results from,

$$\sigma_{rv,t}^2 = \varphi_0 + \varphi_1 \hat{\sigma}_t^2 + v_t \tag{12}$$

4.2 Results

Table 5 reports the out of sample forecasts based on evaluation criteria RMSE, MAE, HRMSE, HMAE and R^2 . The forecasts are based on parameter estimates from rolling samples with fixed sample size of 1264 days.

A number of conclusions may be drawn. First, the GARCH (1,1) model is inferior to other three models: the first order seasonality GARCH (1,1) model, the GARCH (1,1) model extended with range data, and the first order seasonality GARCH model extended with daily range. The daily GARCH (1,1) model has the smallest regression R^2 and highest values for RMSE, MAE, HRMSE and HMAE. Second, the regression based method and summary statistics both suggest that GARCH (1,1) models extended with the difference between daily high and low are better than the use of GARCH models ignoring daily range. Third, including seasonality improves the out of sample forecasts of the daily GARCH (1,1) model. The coefficient of determination R^2, increases from 0.1944 for the GARCH (1,1) model to 0.2051 for the first order seasonality GARCH (1,1) model. Results are qualitatively consistent across four different statistical measures. Fourth, interestingly, the first order seasonality GARCH model extended with daily range is not the best model based on R^2 , MAE and HMAE. The first order seasonality GARCH model extended with daily range has an R^2 0.2233, whereas the GARCH (1,1) model extended with daily range has an R^2 0.2641. The MAE is 0.6511 for the GARCH (1,1) model extended with daily range, whereas it is 0.6569 for the first order seasonality GARCH model extended with daily range. Similarly, the HMAE drops from 0.4737 to 0.4711 when ignoring seasonality. The use of Fourier series does not lead to a superior forecasting performance for the extended GARCH (1,1) models.

5. Conclusion

Previous studies reveal that range-based volatility estimator is highly efficient. However, little attention is paid to the application of these estimators. This paper compares the performance of GARCH models, range based GARCH models, and log-range based ARMA models in terms of their forecasting abilities. The empirical analysis so far makes the following points: For forecasting soybean futures market volatility it is important to include the daily range, defined as the difference between daily high and low. For the extended GARCH models, the adding of seasonality become less important, but it still improves forecasts results in terms of RMSE and HRMSE.

Mean	-0.0097
Standard Deviation	1.3233
Skewness	-0.4045
Kurtosis	6.6273
Minimum	-8.5892
Maximum	5.7397

Q-Test Results				
Lags	Returns		Squared Returns	
	Q-Statistics	P-Value	Q-Statistics	P-Value
Lag 1	0.332	(0.565)	177.27	(0.000)
Lag 2	0.340	(0.844)	229.53	(0.000)
Lag 5	5.936	(0.313)	528.00	(0.000)
Lag 10	12.306	(0.265)	1013.20	(0.000)
Lag 15	19.861	(0.177)	1391.00	(0.000)

Table 1: Summary Statistics for Soybean Futures Returns ($100 \times \ln(P_t / P_{t-1})$)

	Log Absolute Returns	Log Range
Mean	-0.4717	-4.2941
Standard Deviation	1.0154	0.5600
Skewness	-0.5119	0.2405
Kurtosis	3.0743	2.9312
Jarque-Bera Statistics	54.22	12.43
& P-Value	(0.000)	(0.002)
Autocorrelations		
Lag 1	0.170	0.504
Lag 2	0.122	0.488
Lag 5	0.207	0.486
Lag 10	0.155	0.427
Lag 20	0.148	0.392

Table 2: Summary Statistics for Soybean Futures
Log Absolute Returns and Log Range

	No Seasonality		First Order Seasonality		Second Order Seasonality		Third Order Seasonality	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
μ	-0.0167	0.0271	-0.0183	0.0268	-0.0137	0.0273	-0.0105	0.0272
α	***0.0885	0.0150	***0.0786	0.0139	***0.0692	0.0133	***0.0665	0.0125
β	***0.9020	0.0149	***0.9085	0.0151	***0.9158	0.0141	***0.9185	0.0132
κ	**0.0176	0.0079	**0.0220	0.0088	***0.0222	0.0080	***0.0226	0.0079
ϕ_1			0.0003	0.0060	0.0063	0.0062	*0.0104	0.0063
ψ_1			**-0.0170	0.0071	**-0.0137	0.0069	**-0.0171	0.0076
ϕ_2					**-0.0152	0.0064	***-0.0213	0.0076
ψ_2					0.0002	0.0063	-0.0012	0.0064
ϕ_3							*0.0123	0.0074
ψ_3							0.0015	0.0061
AIC		1.5456		1.5436		1.5422		1.5420
SC		1.5537		1.5558		1.5584		1.5624
Log-likelihood		-1949.5908		-1945.0701		-1941.3016		-1939.1201
Skewness		-0.17813		-0.22178		-0.22772		-0.23814
Kurtosis		4.0142		3.9193		3.8482		3.7216
Q-test P-values#								
Lag 1	0.2238	0.5243	0.1350	0.6166	0.0987	0.7622	0.0703	0.7740
Lag 2	0.4769	0.6369	0.3266	0.6930	0.2552	0.5979	0.1937	0.6378
Lag 3	0.2674	0.7768	0.1958	0.7926	0.1896	0.7446	0.1304	0.7989
Lag 5	0.4171	0.1028	0.3461	0.0691	0.3565	0.0492	0.2703	0.0914
Lag 10	0.6260	0.0946	0.5894	0.0787	0.5185	0.0785	0.4280	0.1212

*, **, *** Significant at the 10%, 5%, and 1% level, respectively. # P-values for η and η^2.

Table 3: GARCH Estimation Results

	No Seasonality		First Order Seasonality		Second Order Seasonality		Third Order Seasonality	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
μ	0.0005	0.0270	-0.0003	0.0270	0.0037	0.0276	0.0067	0.0279
α	**0.0547	0.0242	**0.0391	0.0175	*0.0299	0.0159	*0.0262	0.0156
β	***0.8604	0.0376	***0.8641	0.0265	***0.8743	0.0225	***0.8771	0.0229
κ	0.0000	0.0215	0.0000	0.0153	0.0000	0.0143	0.0000	0.0152
Range	***0.0125	0.0040	***0.0144	0.0042	***0.0142	0.0041	***0.0144	0.0045
ϕ_1			-0.0048	0.0087	0.0046	0.0085	0.0083	0.0086
ψ_1			**-0.0236	0.0112	**-0.0249	0.0124	**-0.0289	0.0141
ϕ_2					*-0.0136	0.0082	**-0.0215	0.0105
ψ_2					0.0094	0.0091	0.0116	0.0108
ϕ_3							0.0114	0.0093
ψ_3							-0.0076	0.0086
AIC		1.5374		1.5350		1.5346		1.5348
SC		1.5476		1.5492		1.5529		1.5572
Log-likelihood		-1938.3298		-1933.1845		-1930.7353		-1928.9753
Skewness		-0.14806		-0.18658		-0.19939		-0.20334
Kurtosis		3.7991		3.6683		3.6506		3.5381
Q-test P-values#								
Lag 1	0.1963	0.5978	0.1138	0.6012	0.0966	0.5957	0.0668	0.5842
Lag 2	0.4320	0.5765	0.2861	0.5941	0.2511	0.5064	0.1859	0.4813
Lag 3	0.3017	0.7389	0.2104	0.7500	0.2179	0.6844	0.1530	0.6716
Lag 5	0.4655	0.0141	0.3697	0.0069	0.3776	0.0108	0.2999	0.0188
Lag 10	0.6568	0.0180	0.6150	0.0118	0.5830	0.0210	0.5148	0.0299

*, **, *** Significant at the 10%, 5%, and 1% level, respectively. # P-values for η and η^2.

Table 4: Estimation Results of GARCH Extended with Daily Range

	GARCH (1,1)	First Order Seasonality GARCH (1,1)	GARCH (1,1) Extended with Daily Range	First Order Seasonality GARCH (1,1) Extended with Daily Range
RMSE	1.0183	0.9906	0.8731	0.8718
MAE	0.7122	0.7081	0.6515	0.6569
HRMSE	0.5506	0.5493	0.5474	0.5543
HMAE	0.4765	0.4750	0.4711	0.4737
R^2	0.1944	0.2051	0.2641	0.2233

Table 5: Daily Volatility Forecast Performance

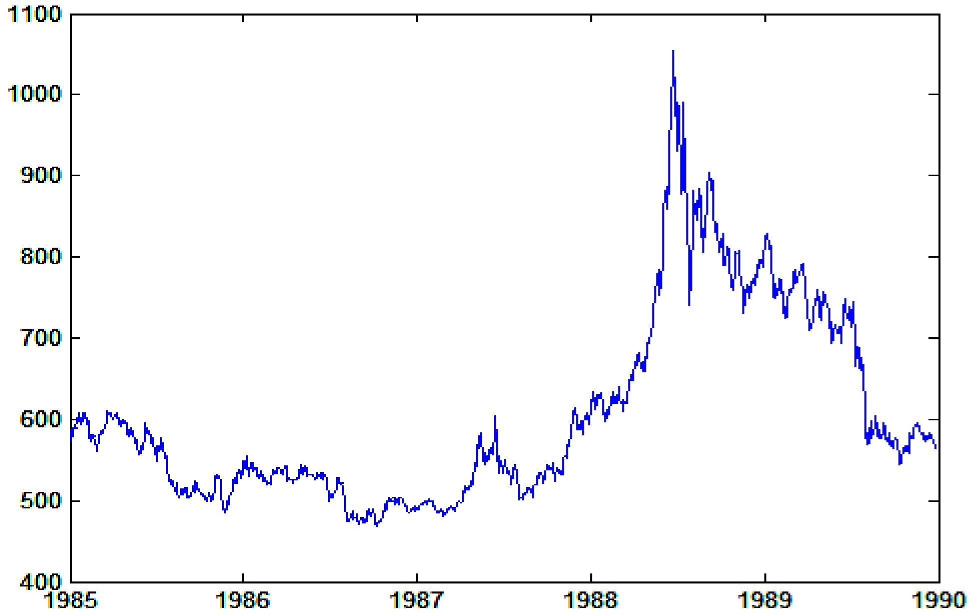


Figure 1: Soybean Futures Prices (1985/01-1989/12)

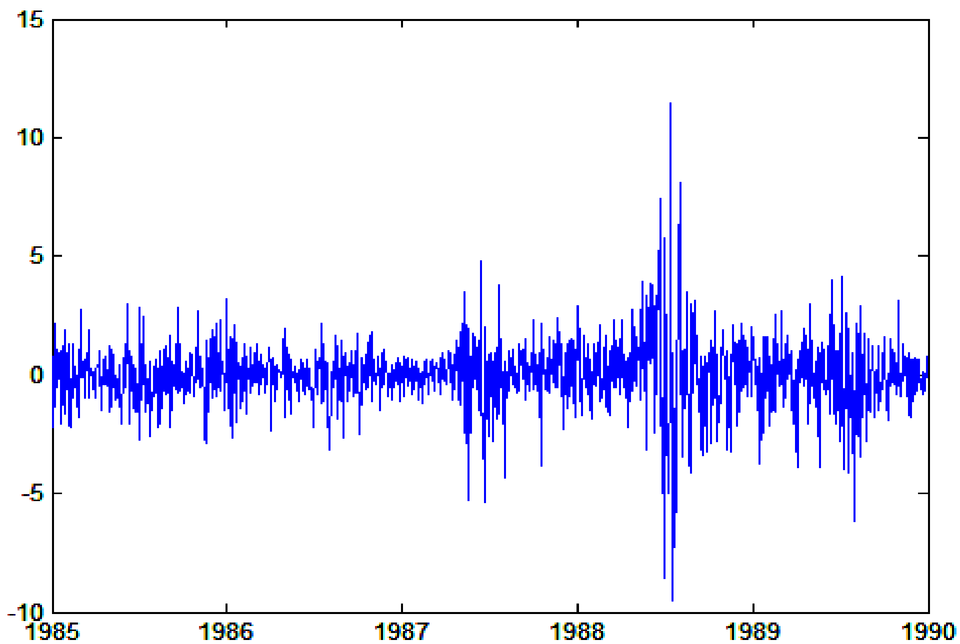
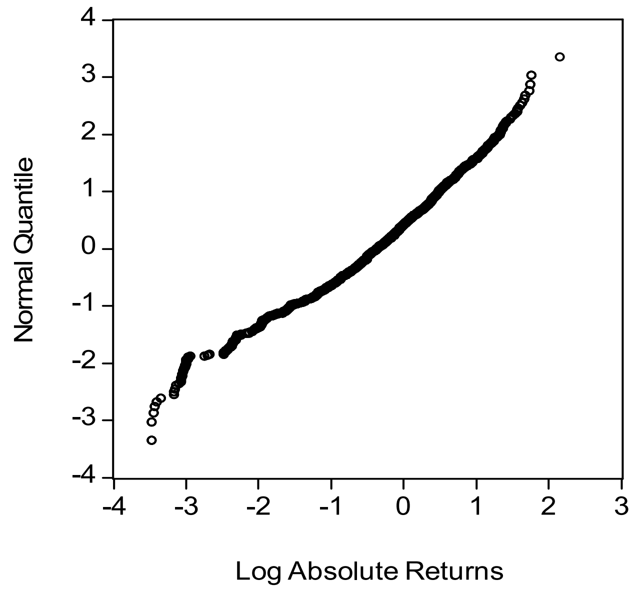


Figure 2: Soybean Futures Returns (1985/01-1989/12)



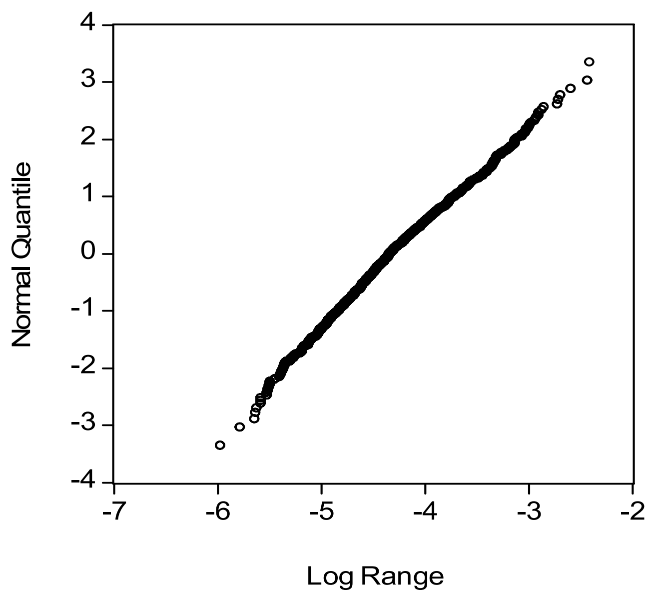


Figure 3: Q-Q Plots of Log Range and Log Absolute Returns

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