Out Of The Box

Examining The Omega Function

Risk Management – Taming The Tail

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Alternative investments, such as hedge funds and private equity, typically have non-normal return distributions typified by fat tails. However, traditional mean-variance approach to measure risk in alternative investments does not distinguish between upside and downside volatility. Ranjan Bhaduri and Bryon Kaneshige explain how the Omega Function does.

(This article is based on a talk given by Ranjan Bhaduri at the 2005 Canadian Annual Derivatives Conference hosted by the Montreal Exchange.)

The importance of proper risk management is a crucial component of the investment process. Financial firms recognize that the spotlight on risk management practices demands, at a minimum, compliance with the Basel Accord and institutional investor requirements.

A good risk management program requires a good characterization of 'risk.' Often, risk is defined as the volatility (measured by the second moment or variance) of an investment or portfolio. Other measurements of risk include beta and Value-at-Risk (VaR). The mathematics of risk is delicate, as there is an entanglement between risk, randomness, and probability. A good chief risk officer will tackle the deep questions and will use proper mathematical tools to measure risk and performance. Investment programs should have a set of risk management beliefs just as they have a set of investment beliefs.

The use of derivatives, when employed correctly, can effectively mitigate certain risk exposures. Each portfolio will have unique needs for such instruments. However, the tool kit of risk and performance measures available is just as important as tuning the risk profile. The focus of this article will be on a particular risk management tool, the Omega Function, which can add value to any risk management program.

A big challenge in any risk management program is dealing with extremely unlikely, but potentially catastrophic, events – managing the tail. One can no longer safely assume normality of the underlying return distribution. One must look at the higher moments, such as skewness and kurtosis. It is here that the Omega Function excels.

It is well-documented that alternative investments (hedge funds, private equity, commodities, real estate) typically have non-normal return distributions typified by fat tails. Moreover, the traditional mean-variance approach does not distinguish between upside and downside volatility. The original motivation for using variance as a measure of risk was the computational ease of the resulting portfolio optimization problem. Since the processing power available today is orders of magnitude greater than that available 50 years ago, why rely on a technique that rests on an assumption that we know to be false?

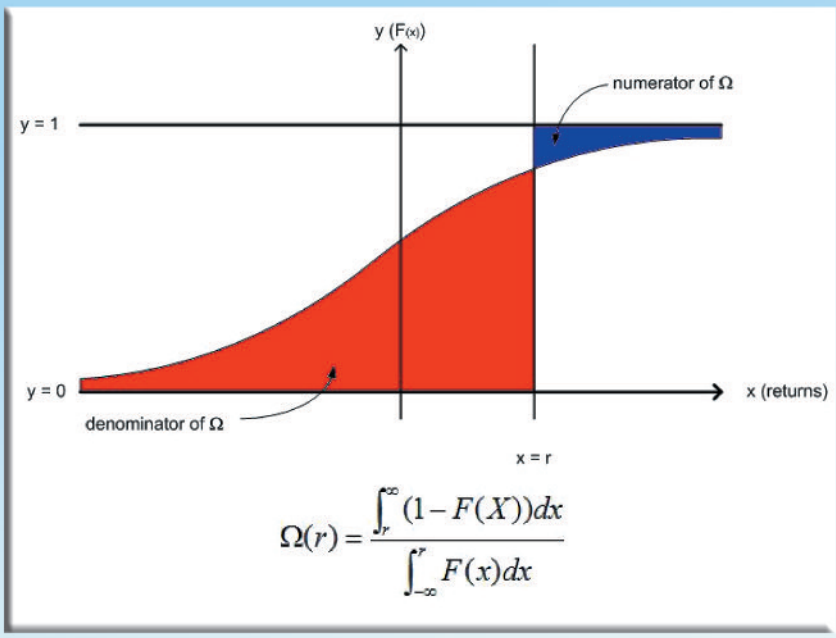
The Omega Function

The Omega Function does not suffer from any of these problems. Invented in 2002 by Con Keating and William Shadwick (a Canadian mathematician), it is similar to the Sharpe Ratio and serves as a ranking function. Unlike the Sharpe Ratio, the Omega Function encodes all of the higher moments and distinguishes between upside and downside volatility. Even William Sharpe, the creator of the Sharpe Ratio, spoke recently criticizing the use of the Sharpe Ratio in investment decisions and lamented the fact that his name is associated with it. The Omega Function has been gaining steam in the investment community and has been referred to as a 'sharper Sharpe.'

The Omega Function is calculated on a threshold which is the minimum return that would be considered a 'gain.' Any return below the threshold is considered a 'loss.' Indeed, while the Sharpe Ratio is calculated once and for all for a given portfolio, the Omega Function is calculated on a range of thresholds. The Omega Function provides much more information than the Sharpe Ratio.

Figure 1

Mathematical Definition of Omega



$$\Omega(r) = \frac{\int_r^{\infty} (1 - F(X))dx}{\int_{-\infty}^{r} F(x)dx}$$

The Omega Function is mathematically equivalent (see *Figure 1*) to the return distribution and has many attractive mathematical properties that furnish the opportunity for proper analysis. Finance scholars Kazemi, Schneeweis, and Gupta have established that the Omega Function captures upside versus downside in a mathematically precise way that has finance intuition. The Omega Function is the ratio of the price of a call to the price of a put with the threshold '*r*' being the strike price of both the call and the put, both of whose underlying asset is the investment's return distribution (see *Equation 1*).

Equation 1

$$\Omega(r) = \frac{C(r)}{P(r)}$$

Consequently, the Omega Function has an elegant and practical foundation in both mathematics and finance. In physics, light exhibits wave-particle duality, a property that has led to many insights. In a somewhat analogous fashion, the complementary definitions of the Omega Function add to its effectiveness.

Plotting threshold versus the corresponding Omega value provides the investment's Omega Graph (see *Figure 2*). Omega Graphs offer yet another, deeper, type of analysis. Omega Graphs make it easy to compare different investments. The intersection point of two Omega Graphs represents the indifference point of an investor. The curvature of the Omega Graph is significant in gaining insight as to the robustness of the investment. Omega Graphs will always be decreasing, but those that decrease at a slower rate will be less sensitive to the threshold, thereby indicating more robustness and more upside juice. Methods of calculus may be employed.

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Omega As A Risk Management Tool

How can the Omega Function be used as a risk management tool?

- Portfolio Construction – The Omega Function can help determine if an investment's risk-return profile fits the investor's goals. Constructing a portfolio that embeds extreme analysis helps with risk management. The best defense is a strong offence.
- Risk Monitoring
- Leverage Setting Tool – Graphical analysis of the family of Omega curves representing different leverages can help determine the appropriate amount of leverage
- Performance Review
- Comparative Studies
- Robustness of the Portfolio – The flatter the Omega Graph, the more robust the portfolio to changes in threshold
- Fine-tuning the Tail

The Omega Function furnishes a quantitative method to do rigorous tail analysis. It can be employed in conjunction with scenario testing and other techniques.

Different Types Of Risks

Obviously, the Omega Function is just one tool in a comprehensive risk management program. There are many different types of risks and the portfolio management team must be on top of the capital markets as seemingly unrelated events may impact the performance of the portfolio.

As always, the first task is to define each risk factor. Nobel laureate Joseph Stiglitz gave the following simple, accurate definition of credit risk: "Credit risk arises from the simple fact that there are an infinite number of people who wish to borrow money, but only a finite number of people capable of paying it back."

Market risk, liquidity risk, operational risk, geo-political risk, criminal/terrorist risk, and model risk are just some other types of risk. Leverage is not a risk per se, but it involves upping the stakes, magnifying both the upside and the downside. If not handled correctly, leverage can lead to blow-ups.

Once a risk factor is defined, appropriate metrics must be constructed to quantify the level of exposure. Liquidity risk is an example highlighting the lack of quantitative tools. Many firms do not have adequate statistics to measure liquidity risk.

One aspect of criminal/terrorist risk is related to computer security. The proper application of mathematical cryptography to secure enterprise data networks can help to alleviate this risk.

Model risk is another underestimated concern. There is a tendency to rely too much on models without a proper understanding of the limitations of the model. If there is not enough data, then clearly any statistical analysis must be taken with a large boulder of salt.

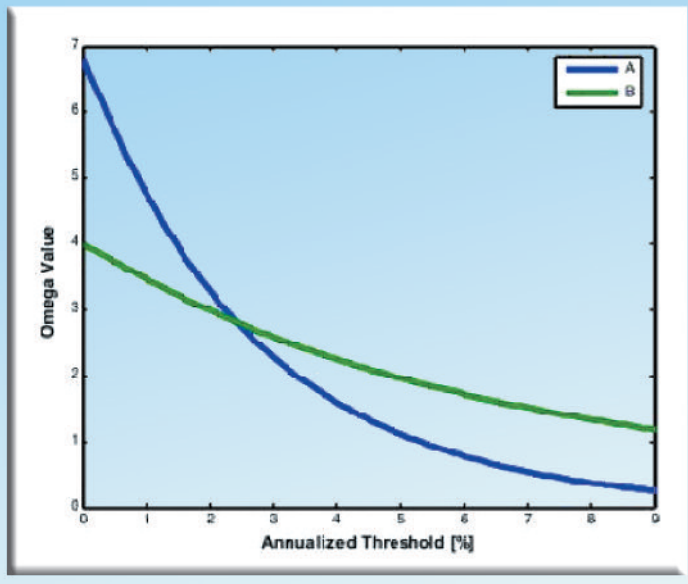
Finally, there is lots of phony stuff being sold as risk management. Do not fall for pretty pictures or be part of the flock. Be tough! How has the model helped in actual investment decisions? When has the model been overridden by qualitative considerations? Has the model been properly vetted and has this been documented? Integrity is key to good risk management and business (where are Arthur Andersen and Enron today?).

A solid risk management program acts in the light of experience guided by intelligence and reason. There should be good quantitative tools that have a solid mathematical foundation, but these tools are no substitute for good judgment and should not be used in an autopilot mode. One needs to do a qualitative overlay on all quantitative methods. Good risk management is alpha. Every good trading strategy is better with good risk management. All of the above are required in taming the tail.

"A strong offence is stronger with a good defense." ■

Figure 2

Omega Graphs



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